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Markets for Information: Of Inefficient Firewalls and Efficient Monopolies*

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Abstract

In this paper we build a formal model to study market environments where information is costly to acquire and is of use also to potential competitors. In such situations a market for information may form, where reports - of unverifiable quality - over the information acquired are sold. A complete characterization of the equilibria of the game is provided. We find that information is acquired when its costs are not too high and in that case it is also sold, though reports are typically noisy. Also, the market for information tends to be a monopoly, and there is typically inefficiency given by underinvestment in information acquisition. Regulatory interventions in the form of firewalls, limiting the access to the sale of information to third parties, uninterested in trading the underlying object, only make the inefficiency worse. On the other hand, efficiency can be attained with a monopolist selling differentiated information, provided entry is blocked. The above findings hold when information has a prevalent horizontal differentiation component. When that is not the case, and the vertical differentiation element is more important, firewalls can in fact be beneficial.

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Keywords: Information sale, Cheap talk, Conflicts of interest, Information acquisition, Chinese walls, Market efficiency.

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1 Introduction

Motivation and Objectives It is common to observe potential competitors in a market exchanging information about issues pertaining to that market. For example, one can see financial analysts providing news over market events, or soccer coaches pondering publicly on the special abilities of (theirs or others') team players. Similar situations arise in the housing market, where we find agents searching and then supplying information over housing properties on sale in the market; in the labor market, where managers often discuss the characteristics of employees in their sector. This is somewhat surprising since the information supplied often has a rival nature. As an example of this rivalry, taken from the private equity market, the following quote from The Economist is illuminating: "Buy-out firms complained that banks which were supposedly advising or lending to them sometimes snatched deals from under their noses. A notorious example was the battle for Warner Chilcott, a British drugmaker, in late 2004: while working with buy-out firms bidding for the company, Credit Suisse teamed up with JPMorgan Chase to launch a bid of its own."[1] This reveals a fundamental conflict arising in information markets. The financial analyst mentioned above may prefer to be the first to use the discovery of an important event, and the soccer coach could be a potential competitor in a bidding war for a particular player. As a consequence, the provider of information may not be trusted to make truthful reports over the information he acquired: the analyst can say that his sources indicate that clinical trials for the wonder drug are going well, when in fact they are flunking.

There is, in fact, a serious concern by regulators about the objectivity and the conflicts of interest faced by financial analysts.[2] This concern is specially important when analysts are not independent, but employed by brokers, who are interested in trading the underlying stocks, as in the case of sell-side analysts, or even by the companies object of study, as in the case of fee-based analysts. Title V of the Sarbanes-Oxley Act, for example, is entirely devoted to "Analysts conflicts of interest."[3]

[1] The Economist, October 12, 2006: "Banks and buy-outs: Follow the money".

[2] Not only regulators, of course. The Economist (July 27, 2007) entitles an article "Analyst for sale. Price: membership of a good club", reporting on an unpublished (even as a working paper) study by James Westphal and Michael Clement in which they survey thousands of analysts and executives. "Almost two-thirds of the analysts admitted to receiving favors from the firms they cover. And those favors appeared to sway their recommendations to their clients... If a company suffers subpar profits, doing a good turn for an analyst cuts the likelihood of a downgrade by half. In the wake of a big acquisition, which analysts tend to frown on, the likelihood falls by 65%."

[3] See also the recent report of the Forum Group (2003) created by the European Commission to deal with

To further understand this problem notice first that in many situations information may be quite costly to acquire. Getting to know that the clinical trial of a particularly promising drug are going well (and thus the stock price of the company doing the research) may require nontrivial effort for the analyst. Similarly, finding out that a young left-footed striker currently playing in a second division Argentinian team is likely to be a star also requires time and money. These costs, together with the fact that information is of common interest, generate a clear incentive for generating a market for information, where the agents who acquired information can provide reports over it, possibly in exchange for the payment of a price, to other agents.

At the same time, the mere existence of information transmission may seem surprising, given the rivalry in its use that we posit. Why would an analyst part with the information about a company rather than make the maximal profit through trading with it? As we will see, this may happen more easily if different individuals have different values for the same bit of information, or if some specific skills or features are needed to profit from a given news. The analyst who finds out about a pharmaceutical company may be specialized in bond-trading, whereas the person with whom he shares the information may be interested in precisely such kind of firms. In the language of industrial organization, we will see that information about a horizontal dimension, instead of a vertical one, is particularly amenable to profitable exchanges. The possibility of exchanging, or selling information to other traders may in turn affect the agents' incentive to acquire information.

The purpose of this paper is to present a model where we can analyze information acquisition and transmission in an environment where the veracity of this information is neither verifiable nor contractible. We are interested in examining when information is acquired; if so, whether a market for information forms, and if it does how it is organized. That is, who and how many traders sell information, who and how many traders buy it, hence how competitive is the market, and how truthful is the information transmitted. But also what are the effects of the information that is transmitted for the performance of the underlying market.

It is also important to find the equilibrium payoffs of the various traders, as this will allow us to evaluate the efficiency properties of equilibria, both the ex-post allocative efficiency of the underlying markets (whether the objects traded end up in the hands of those who value them most), as well as the efficiency of information acquisition. This will allow us to see whether there is scope for regulatory restrictions to improve welfare. In the wake of recent

this problem.

scandals in financial markets, the regulatory bodies of many countries have strengthened the requirements on information dissemination in various markets. One typical recommendation has been to separate who provides information on a market from who trades in it ("firewalls"). For example, section 501, title V in the Sarbanes-Oxley act orders financial firms to establish very specific safeguards to ensure the independence of analysts from traders. Essentially the compensation of analysts should be independent from trading and information flows between the different arms are severely restricted. It also orders these firms to disclose conflicts of interest.[4] It is then particularly of interest to investigate the welfare implication of such restrictions.

The model and results We consider a market where a single, indivisible unit of an object is up for sale. The market is organized as a (second price) auction, where several potential buyers can participate. The good comes in different possible varieties, and each buyer only likes one randomly chosen variety. In addition to buyers, there is the seller, who initially owns the object and has no utility for it, and some other agents who are not interested in trading the object. The true variety is not known ex-ante by anybody, but can be ascertained, incurring a given cost, by any market participant. Besides the market for the commodity there is another market where information is traded: any agent who acquired information can set a price at which he sells a report over his information to other potential buyers. The information transmitted, as we said, is non verifiable, thus reports are pure "cheap talk" messages.

We provide a complete characterization of the equilibria of such game. We find that, when information costs are not too high, information is acquired in equilibrium and in that case it is also sold. That is, the market for information is active. However, the information sold in that market can be noisy: when the provider of information is the seller of the commodity, he tends to hype the value of the good he declares for the agents who purchase information, while when he is a potential buyer, he tends to depress it.

Typically, only one trader acquires information in equilibrium, hence the market for information is a monopoly. Information is either sold at a positive price such that all the uninformed buyers except one purchase it or, when the cost of acquisition of information is low, at a zero price so that all uninformed buyers purchase it. To understand this, notice

[4] The Global Settlement reached on April 28, 2003 between the SEC, the NASD, the NY State Attorney General and NYSE also points in the same direction. Similarly in the report of the European Commission Forum Group (2003), we read: "Analysts' firms should have in place systems and controls to identify and avoid, prevent or manage personal and corporate conflicts of interest."

that the seller of information may benefit even by transmitting information for free as this allows him to manipulate the behavior of uninformed traders in the auction and hence to increase the amount of surplus he can appropriate in the auction.

We also show that, if information is acquired at all, the commodity is allocated efficiently in the auction, i.e. to the agent who values it most. But the level of investment in information acquisition is not efficient, in particular there is typically underinvestment. Interestingly, this inefficiency is present no matter what is the identity of the agent acquiring and selling information, i.e. not only when he is a potential buyer or the seller, but also when he is an agent not interested in trading the object. Actually, in the last case the inefficiency is even more severe. Hence restricting the access to the market for the sale of information only to disinterested traders (as in the case of "firewalls"), while improving the truthfulness of the information transmitted, makes the overall market outcome worse. The reason is that when the seller of information is also interested in trading the object, he has an additional benefit from the information, due to the possibility of trading directly on it. Hence the investment in information is higher. In this respect, it is interesting to note that Boni (2005) and Kolasinsky (2006) document significant decreases in the number of analysts following stocks after the Global Settlement[5] and the passage of Sarbanes-Oxley.[6]

An efficient outcome can be attained if the informed agent can sell different types of reports, of different quality, (or equivalently if we permit the resale of information). We show that in this way the information provider, when he is a potential buyer, can appropriate all the increase in social surplus generated by his information acquisition and dissemination. At the same time, in this case entry in the market for information often becomes profitable. Thus some regulatory intervention may still be needed to have efficiency, protecting monopoly situations in the market for information with barriers to entry, though regulators usually frown upon such practices.[7]

In most of the paper we consider the case we described where the uncertainty concerns the true "variety" of the object up for sale, over which buyers have different preferences. We can thus say, as mentioned above, that information only concerns a horizontal differentiation element. At the end of the paper we consider the case where an element of vertical differen-

[5]See footnote 4.

[6]There is not a clear consensus as to the reasons for this drop (see Parker 2005, Kolasinsky 2006, Zingales 2006). Since that discussion is mostly atheoretical, we believe that our model can shed some light on the mechanisms which underlie this empirical observation.

[7]It even landed a jail sentence to Henry Blodget, the noted Merril Lynch analyst who was issuing an "accumulate" recommendation for Excite at Home while in an internal e-mail he was writing "ATHM is such a piece of crap!"

tiation is introduced: the commodity can now also be of high and low quality, and all buyers prefer, at least weakly, high to low quality. In that case, the degree of truthfulness of the reports transmitted deteriorates, both when the provider is a potential buyer or when he is the seller of the commodity. Hence, if quality is sufficiently important for buyers, firewalls could be welfare improving. The conclusion that emerges is that firewalls are beneficial only in some situations, as the one above, and may thus be imposed with care.

The paper is organized as follows. The environment is described in Section 2 while a complete characterization of the properties of the equilibria and their welfare properties when information providers are potential buyers is given in Section 3. The following Section investigates how the properties of equilibria, in particular their efficiency, vary with other types of providers of information, establishing the adverse effect on welfare of firewalls. Section 5 presents some alternative ways in which the inefficiency problem can be solved. The robustness of our results and some extensions are then discussed in Section 6.

Literature This paper is related to different strands in the literature. More obviously, it is related to the seminal work of Crawford and Sobel (1982) on strategic information transmission. The primary focus of such work and the ensuing literature is the message game and the relationship between information transmission and alignment of the preferences of sender and receiver (or the 'conflict of interest' among them). To that literature, we add a richer game structure. The amount of information available and who 'owns' it are endogenously determined, as a result of the information acquisition decisions of every agent. We also allow messages to be transmitted for the payment of a price, thus formalizing a market for information. And we examine the consequences of the acquisition and transmission of information for the properties of the equilibria in the underlying market for the commodity. Finally, with regard to the message (sub)game, in our set-up the degree of coincidence of the objectives of sender and receivers is not common knowledge, as it depends on the realization of the true variety of the object and of the preferred variety of each potential buyer, which is only privately known to him.

There is also a rather large empirical literature which studies the behavior of financial analysts, and in particular the presence of biases in their reports, and its effects for the performance of asset markets; e.g., Womack (1996), Michaely and Womack (1999), Barber et al. (2001), Agrawal and Chen (2006), Bradshaw, Richardson and Sloan (2003), Jegadeesh et al. (2004), and the recent survey by Mehran and Stulz (2007).

On the other hand there is much less theoretical work on markets for information. A good part of the attention has received the case where the quality of the information transmitted is perfectly verifiable, thus abstracting from the problem of untruthful reports, as well as from the problem of information acquisition. Admati and Pfleiderer (1986, 1990) look at a situation where market participants act as price takers, where the "paradox" arises that when information is too precise, asset prices are perfectly revealing, so that information is worthless. Therefore, providers need to add some noise in order to profit from information sales.[8] When traders are strategic, information transmission may also provide a strategic advantage, as pointed out by Vives (1990) in a general oligopoly framework, and Fishman and Hagerty (1995) in the case of financial markets.

The case where the information transmitted is non verifiable, as in our set-up, has been considered by Morgan and Stocken (2003), who study the information transmitted by an analyst when his incentives may not be aligned with those of investors, as he may be either a type that enjoys higher utility when the price of the underlying asset is high, or a type that enjoys telling the truth. They find that the analyst always "hypes" the stock; see also Kartik, Ottaviani and Squintani (2007). This is in line with our results for the case in which the information provider is the seller of the object. Bolton, Freixas and Shapiro (2007) study how a cost for lying and competition can mitigate the tendency of financial intermediaries to sell to their costumers products that are not appropriate for their tastes. They share with our work the feature that the information transmitted has a horizontal dimension, but they focus on the incentives for truth-telling by sellers who may sometimes carry all existing varieties and some other times only one.

Our analysis, being cast in a static framework, abstracts from reputational concerns. These may arise in a dynamic framework, where providers of information and traders repeatedly interact, and may mitigate the tendency of providers to send untruthful reports which may damage their future reputation, as shown by Benabou and Laroque (1992), and Ottaviani and Sorensen (2006).

2 Model

There is one object for sale, initially owned by an agent, indicated as the seller of the object, who has no utility for it. The type of the object is uncertain: there are K possible varieties, all with the same ex-ante probability. Let the true type of the object be $v \in \mathcal{K} = \{1, 2, ..., K\}$. There are then N potential buyers, agents who may be interested in purchasing the object.

[8] A similar point is made in Milgrom (1981) who explores how to inform others, if providing too good information can hurt the provider.

We denote buyer $i \in \mathcal{N} = \{1, ..., N\}$ by B_i; such buyer only cares (has positive utility) for one particular variety in $\mathcal{K}$ indicated as θ_i. The variables $\{\theta_i\}_{i \in \mathcal{N}}$ and v are all i.i.d. over $\mathcal{K}$, thus for all $i, j \in \mathcal{N}$, θ_i is uncorrelated with θ_j and v; all elements of $\mathcal{K}$ have then the same probability, $1/K$. The object is allocated to buyers via a second price auction.

Information structure. The realization of θ_i is private information of individual i. On the other hand, the type of the object for sale is not known to any trader. Before the auction takes place, anybody can acquire, by paying a cost c, a signal over the type of the object, which we assume is perfectly informative. If a trader acquires such signal he can in turn 'sell information' to other traders.

The utility of buyer B_i can then be written as

$$\pi_{B_i} = I_v - cI_e - t_{B_i},$$

where t_{B_i} is the sum of the net monetary payments made by B_i to the seller to gain possession of the object and/or to the other traders to purchase or sell information to them. I_v is an indicator variable that takes the value 1 if B_i gains the object and $v = \theta_i$ (i.e., the object is of the type B_i likes) and 0 otherwise. Finally I_e is another indicator that takes the value 1 if B_i decides to acquire the signal over the type of the object, and 0 otherwise.

In this paper we are interested in situations where the information sold is not verifiable, i.e. the seller of information sends a report, which is pure 'cheap talk', over the signal he received. Since we abstract from reputational concerns (there is a single period), information is transmitted only when the seller cannot profit by distorting his report. This in turn requires us to examine the consequences of transmitting false or truthful information for the behavior of buyers in the market for the good. In the environment described, different buyers might, but also might not, be interested in the same type of object, the uncertainty only concerns a horizontal differentiation component of the object. Hence the information transmitted, while being rival, has also an element of non-rivalry. One could interpret the specification of the model as capturing situations where the agent who sells information is not always able to profit directly from the information acquired. For example, leveraging the information may require the possession of complementary assets or skills, which he may lack. In section 6 we extend the model to allow also for the presence of a vertical differentiation component.

We examine first the case where information can be acquired and transmitted by potential buyers of the object. The possibility that information is sold by other traders, as the seller of the object and/or agents not interested in trading the object, is considered later, in section

4. Furthermore, we assume that each seller of information sells a single, identical report to all buyers, at the same price; we refer to this situation as no differentiation of the quality of the information sold. In section 5.1 we discuss the case where different kinds of reports may be sold by the same trader, at different prices.

Timing of the game.

1. First, each potential buyer decides whether or not to acquire the signal over the type of the object. The cost of the signal is c. The decision to acquire information, but obviously not the information itself, is commonly observable by all agents.

2. Any potential buyer who has chosen to acquire information, before learning the realization of the (perfectly informative) signal over the type of the object, can post a price p at which he is willing to sell a report over the signal, which will be sent after receiving it.[9]

3. Each of the buyers who did not choose to acquire information in stage 1. decides whether or not to purchase information from any of the traders selling information (possibly from more than one seller). Each of these buyers has then a final chance, after the market for information closes, to acquire the signal at a cost of c.[10]

4. All agents who paid the cost c of information acquisition learn the realization of the signal. They then issue a report to the buyers who purchased information from them.

5. A second price auction takes place among all the buyers for allocating the object.

Message subgame. We consider the case where the set of messages available to a seller of information is the set of direct messages:

$$\mathcal{M} = \{1, 2, ..., K\},$$

[9] Thus the price posted has no signaling content. In section 6 we discuss the effects of considering the case where the price is posted only after learning the realization of the signal, as well as alternative auction formats, and other robustness checks.

[10] This last opportunity of direct information acquisition, with no opportunity to resell it, limits the ability of the sellers of information to corner the market and extract surplus from buyers. The timing reflects the fact that acquiring information entails a simpler technology and takes less time than organizing a market for selling reports over it.

i.e. it coincides with the set $\mathcal{K}$ of possible types of the object. The report sent by the seller is then given by one of the K possible types of the object. Moreover, the message structure is augmented so as to involve two phases:

a. Each uninformed buyer who has agreed to purchase information sends first a report over his type to the trader selling information (the set of available messages for an uninformed buyer B_i is again the set of possible types of the buyer, given by $\mathcal{K}$). Such report is observed only by the seller of information, not by the other buyers.

b. Subsequently, the seller of information sends a report over what he learned to all the buyers who purchased information from him.

It will be clear from the analysis in the next Section that the presence of the first phase, by providing the seller of information with some information over buyers' preferences, allows to enhance his revenue and, furthermore, that there is no essential loss of generality in restricting attention to direct messages.[11]

We will assume that the number of buyers is not 'too small' - in particular that N is at least larger than 3 - so as to avoid the trivial case where there is no competition among buyers. In addition, we consider the case where the number K of possible types of the object is strictly greater than the number N of potential buyers:

$$K > N,$$

so that the competition among buyers for the object, while always present, is not very intense.[12]

3 Equilibrium and welfare

3.1 Equilibrium

Since information transmission needs not be truthful, the game (and in particular the message subgame, starting from stage 4 of the game) has many equilibria, as is common in other kinds of "cheap talk" games (see Crawford and Sobel 1982). We focus our attention on the

[11] We will show in particular that our analysis and results extend to the case where the set of messages is expanded to include the possibility of sending no report.

[12] The only role of this condition is to simplify the presentation. As argued in footnote 14, our results remain valid when $K \leq N$.

equilibria where the degree of truthfulness in agents' reporting - and hence the revenue of the seller of information - is maximal.

We will show that an equilibrium always exists where uninformed buyers truthfully report their type to the seller of information and this one adopts the following reporting strategy (both in and out of equilibrium):

$$m_i = \begin{cases} v, \text{ if } v \neq \theta_i & \\ y, \begin{cases} \text{with probability } \frac{1}{K - N(B_i)}, \\ \text{for all } y \neq \theta_j \; \forall B_j \in \mathcal{N}(B_i), \end{cases} & \text{if } v = \theta_i \end{cases} \tag{1}$$

where B_i denotes the buyer selling information, m_i is the report issued by him, $\mathcal{N}(B_i)$ the set given by all the buyers purchasing information from B_i, and $N(B_i)$ the number of distinct realizations of θ_j across all buyers $B_j \in \mathcal{N}(B_i)$. Therefore, trader B_i tells the truth about the type of the object when the true variety of the object does not coincide with his own type (i.e. with the variety he likes). On the other hand, when the two coincide B_i faces a conflict of interest as he wishes to get the good at the lowest possible price, and this price generally depends on the report sent by him. Thus, he will not tell the truth and will send a message which is a randomization over all the types which are different from the type of all the agents buying information from him.[13] One could interpret this message as telling the buyers of information that the object is not appropriate for any of them.[14]

It should be clear, also from the following analysis, that the reporting strategy described in (1) entails the maximal degree of information transmission at an equilibrium. Since there is no cost for lying, the seller is only willing to tell the truth when he cannot gain a strictly higher payoff in the market (i.e. in the auction) by lying and this happens when he is not interested in the object. If the seller's reports are informative and hence affect buyers' beliefs and bids, we should expect a buyer, upon receipt of a report saying that he likes the object, to raise the belief that he indeed likes the object and hence his bid, and decrease them otherwise. Hence when the seller is interested in getting the object he wants to deceive buyers and send a report (as the one above) telling them they do not like the object, which

[13] Strictly speaking, when the seller likes the object but no buyer of information likes it, the seller might still tell the truth (as in this case it would have no negative effect for him).

[14] When $K \leq N$ the set of available messages given by $\mathcal{M}$ is clearly not rich enough to obtain this outcome. However, this could be achieved by enlarging the set of possible messages to allow for the possibility that a separate message is sent to each buyer of information, telling him whether or not he likes the object. With this extension, the properties of the equilibrium outcome found in the next section remain valid also when $K \leq N$.

induces them to lower their bids and hence the price at which the object is gained by the seller of information

An additional source of multiplicity of equilibria comes from traders' behavior in the auction, in the final stage of the game. We show that there is always an equilibrium of the auction where the bid of each trader equals his expected value of the object, conditional on winning the auction, and focus our attention on such equilibrium ('with truthful bidding'). Other equilibria may exist, but are typically non robust to trembles and we will ignore them in what follows.

We will characterize the perfect bayesian equilibria of the game described in the previous section and evaluate their welfare properties for different parameter configurations (in particular, for different levels of the cost of information acquisition, c). Given the selection of equilibria in the message and auction subgames specified above (quite natural, we would like to argue, given our purposes), we will show that the overall equilibrium is, for almost all parameter values, unique:

THEOREM **1** *For all $c \geq 0$ there exists a perfect bayesian equilibrium of the game with no differentiation of the quality of information sold where sellers of information adopt the reporting strategy in (1), buyers of information truthfully report their type to sellers, and participants in the auction adopt a truthful bidding strategy. Furthermore:*

1. *If $c \geq c^I \equiv \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, no buyer chooses to acquire information; the object is then gained by a randomly chosen buyer, at a price $1/K$.*

2. *If $c^I \geq c \geq c^0 \equiv \frac{1}{K^2}\left(\frac{1}{N-2}\right)$, one buyer acquires information and sells a report over it at a price $p = \min\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, c\right\}$, at which all the other buyers except one purchase information; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to $1/K$ (when either the seller of information, or only one buyer of information likes the object), 0 (when neither the seller nor any buyer of information likes the object) and 1 otherwise.*

3. *If $c^0 \geq c$, one buyer acquires information and sells a report over it at a price $p = 0$, at which all the other buyers purchase it; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price equal to 0 (when the seller of information, or exactly one of the other buyers, who all purchase information, likes the object), and 1 otherwise.*

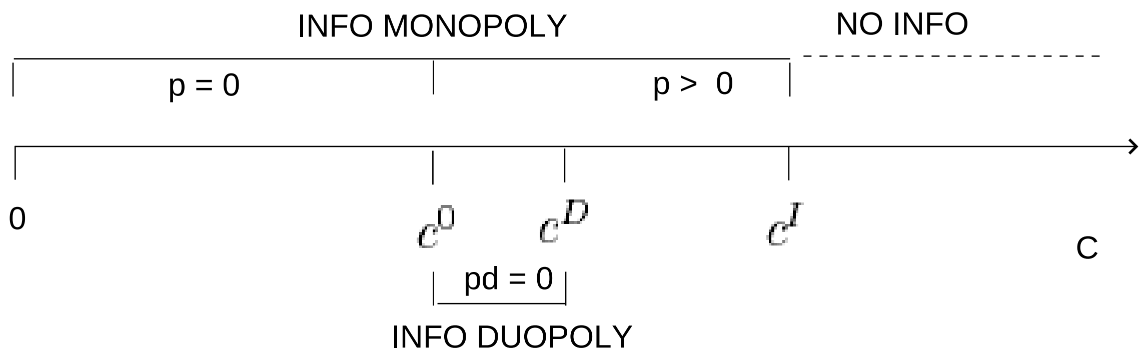

Figure 1: Equilibrium with homogeneous messages

This is the unique equilibrium outcome with reporting strategies exhibiting maximal degree of truthfulness and truthful bidding strategies for all possible values of c*, with the only exception of a subset of region 2., given by* $c^D \equiv \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq c \geq c^0$*, where another equilibrium exists, with two buyers acquiring information and each of them selling a report over it to all other buyers, at a price* $p = 0$*; the object is then always gained by a buyer who likes it, if such a buyer exists, at a price of* 0 *if nobody else likes it, and* 1 *otherwise.*

Thus when information costs are low enough, information is acquired in equilibrium. Whenever it is acquired, information is transmitted via a report that in some events is truthful while in others is a lie. Information is sold for a low enough price so that all buyers, or all buyers except one, purchase it. The market for information is typically a monopoly. Furthermore, the seller of information always gets the object when he likes it; when he does not like it, the object goes to one of the buyers of information who likes it, if such buyer exists and otherwise, in case 2. goes to the buyer not purchasing information, while in case 3. goes to a random buyer who purchased information. Figure 1 summarizes the result.

3.2 Proof of Theorem 1

We present here the main steps of the proof of the Theorem while leaving some details in the Appendix. First, the consistent beliefs of buyers associated with the sellers' reporting strategy in equation (1) are determined. Then we study the traders' strategies in each stage of the game, and establish their optimality given the beliefs.

Beliefs With the message structure in (1) there are no out-of-equilibrium messages. Thus, we can find the beliefs for an uninformed buyer, say buyer B_j, who receives a report from an informed buyer, say buyer B_i, using Bayes' rule in all cases:

- When buyer B_j receives from B_i a message $m_i = \theta_j$ he knows for sure that he likes the object (the message is truthful). That is, $\Pr(v = \theta_j | m_i = \theta_j) = 1$.

- When buyer B_j receives a message different from his type, this may happen for two reasons: either the message is truthful, and then B_j does not like the object, or it is not truthful, as the sender likes the object and is randomizing over types different from his own and the types of buyers of information. In this case there is then a positive probability that B_j may like the object.

Finally, the buyers who neither acquired information directly, nor indirectly by purchasing it in the market, have beliefs equal to their prior beliefs. That is, $\Pr(v = \theta_j) = 1/K$. The beliefs of a buyer who is purchasing two (or more) distinct reports from two (or more) informed buyers are similar.

Stage 5: Behavior in the auction

Given the beliefs of buyers who purchased one report we can show that a 'truthful bidding strategy' - where a trader's bid equals his expected value of the object, conditional on winning the auction - is always optimal if all other traders adopt such strategy:

- When a buyer receives a message indicating that the object is of the type he likes, his belief - as argued above - is that with probability 1 he likes the object. His optimal bid when the other bidders adopt 'truthful bidding strategies' is equal to 1, i.e. to his posterior beliefs about the valuation of the object, and is then also truthful.

- A message different from the type a buyer likes is received as we said in two alternative events. The first one is when the buyer likes the object, but so does the sender of the message. In that case the buyer cannot win the object in the auction with a positive surplus, i.e. at a price lower than his valuation. This is because the seller of information is better informed and, if he adopts a truthful bidding strategy, will make a higher bid, equal to 1. The other possibility is that the buyer does not like the object, in which case any positive bid, if successful, would yield him a negative surplus (with positive probability). Thus any positive bid yields a negative expected payoff. The optimal bid of a buyer of information, after receiving a message different from his type, is then equal to zero, which is his expected value of the object conditional on winning the auction; so in this case too it is a truthful bidding strategy.

Notice that, even though we are in a second price auction the buyer's bid is not always equal to his posterior belief over the value of the object. This is due to the correlation of the information of traders induced by the sender's reporting strategy. The information conveyed by winning the auction should then also be taken into account.

In contrast, the buyers who do not purchase any information do not suffer from a correlation problem (there is no relevant information for them in the event of winning the auction). Thus, the optimal bid in their case is: $\Pr(v = \theta_j) = 1/K$.

How important is the restriction to 'truthful bidding strategies'? We claim such restriction only bites when there is a single trader who is not acquiring information nor purchasing it from other buyers, i.e. who chooses to remain uninformed. In this case any strictly positive bid lower than $1/K$ by the uninformed buyer gives him the same payoff as a bid of $1/K$; this may in turn affect the other traders' optimal bidding strategies and generate other equilibria. On the other hand, when more than one trader remains uninformed, or all traders purchase information, the only equilibrium in the auction subgame is the one with truthful bidding strategies. We can argue therefore that the equilibria where not all bidders follow a truthful bidding strategy, if they exist, are non robust to trembles concerning the decision of purchasing information.

The bidding behavior of an uninformed buyer receiving more than one report from different informed buyers is analogous to the one described above. When his posterior belief over the value of the object equals one, he will bid one, and will bid zero otherwise (when the messages received are not fully informative).

Stage 4: Behavior in the message subgame

We show first that the reporting strategy we postulated for a seller of information is indeed optimal for such trader. We then verify that each buyer of information is willing to report truthfully his type to the seller of information. A key element in the argument is that, by changing the message strategy, neither the seller nor any buyer of information can affect the outcome of the auction in his favor.

Seller: There are two possible deviations which need to be considered for the seller of information. When he likes the object, he may deviate and announce a type corresponding to one of the traders purchasing information from him. If he does that, the bid of that trader will be higher and equal to 1, and the seller will end up paying more for the object than if he had followed the equilibrium message strategy, so he never wants to make such deviation. Second, when the seller does not like the object, he may deviate by announcing a type different from the true one. But that only changes the outcome in the auction, which has no effect on the seller's utility in this case since he is not interested in the object. So the seller does not gain with such a deviation either.

Buyer: A deviation by a buyer of information consists in reporting something different from his true type. This has no consequence when the seller of information does not like the object, because in that case the seller reports the truth, no matter what are the reports he receives from the buyers of information. On the other hand, when the seller of information likes the object a buyer's lie may change the seller's report; in particular, it may induce the seller to announce the buyer's type (both when this is equal and when it differs from the true type of the object). However, in this second case the seller of information always bids 1, hence the buyer still cannot gain any surplus. It thus follows that the buyer of information cannot gain, and may actually lose, by misreporting his type.

Stage 3: Purchase of information.

In this stage each uninformed buyer has to choose whether to purchase information from one - or more - of the informed buyers who are selling information, at the price posted by them, or alternatively to acquire the information directly (at the cost c), or do nothing of the two.

Stage 2: Sale of information.

This is the key stage of the game, where the market for information opens and each trader who at stage 1 has chosen to acquire information posts a price at which he is willing to sell a report over it to any other buyer. The price is set at the level which maximizes the utility of the seller of information, i.e. his expected payoff in the auction plus the revenue from the sale of information, taking as given the strategies of the other sellers, if any, and the response strategies of buyers in the next stage to the prices posted. The main elements of the analysis are summarized below.

The **maximal willingness to pay for information** of an uninformed buyer is given by the amount by which the buyer's payoff in the auction increases if he purchases information and hence becomes indirectly informed, relative to the best of his two alternatives: acquire information directly at the cost c, or remain uninformed. The expected payoff in the auction of the buyer of information is in turn determined by the probability that he gains the object with a positive surplus, which occurs when he likes it and no other trader who is directly or indirectly informed likes it, and the price at which the object is gained. Letting J be the total number of buyers who did not acquire information, either directly or indirectly, the probability of this event is $1/K\left[(K-1)/K\right]^{N-J-1}$, and is clearly higher the lower is the number of agents who are purchasing information. The price paid to win the object in the auction in this event, given the bidding strategies described in the previous steps, is 0 if $J=0$, i.e. if no buyer remains uninformed, and $1/K$ otherwise.

If an uninformed buyer were not to purchase information but to acquire it directly in the next stage of the game, his payoff in the auction would be exactly the same, minus the cost c. This implies that, for the sale of information to take place, its price p cannot be higher than c.[15] On the other hand, if the buyer remains uninformed his payoff is zero if there are other uninformed buyers, while if he is the only uninformed trader it is positive and equal to $1/K\left[(K-1)/K\right]^{N-1}$. By comparing this expression with the ones obtained above and in the previous paragraph, it is immediate to see that a buyer is only willing to pay a positive price for information when not all the other buyers are either directly or indirectly informed ($J \geqslant 1$). In this case (i.e. when $J \geqslant 1$), the maximal price he is willing to pay is:

$$\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\right\},$$

which is strictly decreasing in $N-J$. Thus the demand for information is strictly decreasing in its price p, for $p<c$.

When there is a **monopolist seller of information**, he always sets the price so as to extract all the surplus from the number of buyers he wishes to attract, i.e. equal to their maximal willingness to pay. The issue is then to determine the number of buyers of information which maximizes the seller's payoff. In the Appendix, we show the following:

Claim **1** *The* ***revenue from the sale of information*** *for a monopolist seller of information is maximized by setting the price p low enough that all uninformed buyers, except one, purchase information, i.e. such that $J=1$.*

The second component in the payoff of the seller of information is given by his **payoff in the auction.** Given the reporting and bidding strategies described above, the sale of information has no influence on the fact that a monopolist seller always gains the object whenever he likes it. Hence its only possible consequence is on the price at which the seller gains the object in the auction, via its effect on buyers' bids. As argued above, when the

[15] Also, if $p<c$ no buyer chooses to acquire information directly in stage 3 of the game.

seller sets the price of information low enough - at zero in fact - that all uninformed buyers purchase information ($J = 0$), he gains the object at a zero price, otherwise he has to pay $1/K$ to get the object (the same as with no sale of information).

This leads us to the **crucial tradeoff** faced by the monopolist seller. He has in fact to choose between the price which maximizes his revenue from the sale of information (and induces all uninformed buyers except one to purchase information) and the lower (zero as we said) price which maximizes his payoff in the auction, attracting all buyers. Then, as we show in the Appendix:

CLAIM **2** *When $c < c^0$ the* ***payoff of the seller of information*** *is maximal if he gives away the information for free, i.e. sets $p = 0$, to guarantee that the auction price is low. On the other hand, for $c > c^0$ the price at which information can be sold is sufficiently high that the seller's payoff is maximal with $p > 0$ ($J = 1$).*

With an **information oligopoly,** the equilibrium price of information is always zero**.** Note first that, when there are two or more sellers of information, the additional benefit for an uninformed buyer of purchasing a second report is always zero. This follows from the fact that purchasing information from one seller allows to gain a positive surplus in the auction only when the buyer likes the object and nobody else who is informed, either directly or indirectly, likes the object. Purchasing information also from a second seller allows the buyer to have more precise information in the event in which one of the two sellers of information likes the object (since the other tells the truth); however in such event no positive surplus can be gained since the seller who likes the object bids one.

Furthermore, the benefit for a buyer of purchasing one report is essentially the same as when there is a monopolist seller; in particular, it is positive only if not all the other buyers purchase information, i.e. buy at least one report. Given that each buyer is willing to pay a positive price only for one signal, and only if not all other buyers purchase information, the only possible equilibrium with positive prices would entail a split of the buyers between the different providers of information, with at least one buyer not purchasing information. But then each of the sellers would have an incentive to undercut. By lowering his price the seller would retain all those already buying from him and manage to steal the buyers from the other sellers of information. This produces a discrete jump not only in his revenue from the sale of information but also in his payoff in the auction; the latter is in fact positive (and equal to $1 - 1/K$ if at least one trader is not purchasing information) when the seller likes the object and neither the other sellers of information, nor any other buyer that is purchasing information from the *other* sellers, likes the object. Hence the probability that a seller has a positive surplus increases with the number of buyers who purchase information only from him. Since such incentive to undercut persists as long as the posted prices for information are positive, the only possible equilibrium obtains when all sellers post a zero price for information and all uninformed buyers purchase information from every seller. In this situation, the sellers of information have the same payoff as the buyers of information, less the cost of acquiring information, thus their overall payoff is lower.

Stage 1: Information acquisition

Having determined the benefits for a buyer of acquiring information, we immediately find when this is profitable:

CLAIM **3** *When the cost of acquiring information is so high ($c \geq c^I$) that it exceeds the maximal gains that a monopolist seller of information can get from the sale of information ($[(N-2)/K][(K-1)/K]^{N-1}$) plus the gains from obtaining the object in the auction ($(1/K)[(K-1)/K]$), no buyer chooses to acquire information. On the other hand, when $c \leq c^I$ one buyer always acquires information.*

As argued above in the discussion of Step 2, when a buyer acquires information he always chooses to sell a report over it. Furthermore, the entry in the market for the sale of information by a second buyer is never strictly profitable, as the payoff of an informed duopolist is less or equal than the payoff that trader would get if he did not acquire information directly but rather purchase it from the other informed buyer. We show in the Appendix that

CLAIM **4** *For a range of intermediate values of c, $c^D \geq c \geq c^0$,[16] the payoff is exactly equal for an informed duopolist and a buyer of information with a monopolist seller of information. In this case there are two equilibria, one with a monopolist seller of information and the other with two sellers of information. Outside this range there is a unique equilibrium with a monopolist seller.*

3.3 Welfare

We now discuss the welfare properties of the equilibria described in the previous section. In particular we are interested in comparing the equilibria to the Pareto efficient allocations, i.e. to the allocations which could be attained by a planner who (knows the buyers' types

[16]For c in this interval, a monopolist seller sets the price of information at $p = c$.

but) is also uninformed about the type of the object and may acquire information, at the same cost c, over it. Given the assumed transferable property of traders' utilities, welfare can be simply evaluated by considering the total surplus, or the sum of the payoffs of all buyers and the seller of the object.

Notice first that, if information is acquired by some buyer, the resulting equilibrium allocation is always ex post efficient, in the sense that the object always goes to a buyer who likes it the most. Hence the only possible source of inefficiency may lie in the information acquisition decision: is that also efficient at equilibrium, or rather is there overinvestment, or underinvestment in information? Evidently, the equilibrium with two buyers both acquiring information is always inefficient as the duplication of the investment in information acquisition is always wasteful. On the other hand, at an equilibrium where only one buyer acquires information there is no wasteful duplication. Such equilibrium was shown to exist for all $c \leq c^I$, while for $c > c^I$ no information is acquired.

To assess the efficiency of such equilibrium we need then to find the threshold for information to be acquired at an efficient allocation and compare it to c^I. If information is acquired, the object can always be allocated to a buyer who likes it, when such buyer exists. In that event the total surplus of traders from the object equals one, while it is zero otherwise. Total welfare is then obtained by subtracting the cost of information:

$$W_1 = P(\exists i | v = \theta_i) - c = 1 - \left(\frac{K-1}{K}\right)^N - c.$$

On the other hand, if information is not acquired the total surplus is one only if the agent who receives the object (and, with no information, this agent can only be randomly chosen) happens to like it. Thus total welfare is in that case:

$$W_0 = \frac{1}{K}.$$

By comparing W_0 and W_1 we find that it is socially efficient for information acquisition to take place if, and only if, $1 - ((K-1)/K)^N - c \geq 1/K$, or:

$$\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \geq c. \tag{2}$$

We show in what follows that this threshold is lower than c^I:

Proposition 1 *In equilibrium there is a less than efficient level of investment in information. In particular, for values of c lying in the following, non empty interval:*

$$c^I < c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) \tag{3}$$

no information is acquired in equilibrium, though it would be socially efficient to acquire it.[17]

Thus there is a range of values of c for which acquiring information is efficient but in equilibrium the gains from information acquisition are too low so that nobody chooses to become informed. To understand the reasons for this result, it is useful to examine first the distribution of the welfare gains and losses across agents when we compare the situation where no information is acquired to the equilibrium with a monopolist seller of information, in particular when c is below but close to its threshold value c^I.

Who gains and who loses from information acquisition When $c \in [c^I, c^0]$ in equilibrium there is one buyer, say B_1, who acquires information directly and then sells it, as a monopolist, and another buyer, say B_N, who remains uninformed. B_1 clearly gains, with respect to the situation where no information is acquired, as his payoff goes from 0 to a strictly positive level (except when $c = c^I$); so does B_N, whose payoff[18]

$$\pi_{B_N} = \frac{1}{K}\left[\frac{K-1}{K}\right]^{N-1} \tag{4}$$

is strictly positive. On the other hand, the payoff of the remaining buyers, who acquire information indirectly by purchasing a report in the market, is unchanged at zero when c is smaller but close to c^I.

What about the seller of the object? His payoff, in the region under consideration is given by

$$\left[\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}-(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right]+\frac{1}{K}\left[\frac{1}{K}+(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\right]$$

As claimed in 2. of Theorem 1, the price at which the object is gained in the auction can be 1, $1/K$ or 0. The terms in square brackets above are then the probabilities of the auction price being, respectively, 1 and $1/K$. The difference in the revenue of the seller of the object between this case and the one where nobody is informed (where the price in the auction is always $1/K$) is then:

$$\Delta\pi_S = \left(1-\frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}-(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right]-\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}, \tag{5}$$

[17] The proof of this and the following propositions are in the Appendix.

[18] See equation (16) in the Appendix.

which is positive if, and only if:

$$1 > \left(\frac{K-1}{K}\right)^{N-3}\left(\frac{K+N-2}{K}\right) \tag{6}$$

As we show in section 4.2 below, this inequality is satisfied for some, but not all admissible values of K and N.

The source of the inefficiency From the ex post efficiency of the equilibrium allocations with a monopolist seller of information it follows that the sum of the changes in the payoff of all traders between the equilibrium with and without information acquisition equals the difference between the levels of maximal total welfare in these two situations, $W_1 - W_0$. The analysis of the distribution of the welfare changes across agents in the previous section allows us so to gain some further understanding of the source of the inefficiency result we obtained. Since, as we said, the payoff of the indirectly informed buyers is zero in both situations (when c is close to c^I), the change in total welfare $W_1 - W_0$ equals the change in the payoff of the seller of the object $\Delta\pi_S$ plus the payoff of the buyer who acquires and sells information and the payoff of the buyer who remains uninformed:

$$W_1 - W_0 = \Delta\pi_S + \pi_{B_1} + \pi_{B_N}. \tag{7}$$

Underinvestment in information obtains if $\pi_{B_N} + \Delta\pi_S > 0$, i.e. if the trader acquiring information is unable to recoup all the gains in social surplus generated by his decision. From (4) and (5) we get:

$$\pi_{B_N} + \Delta\pi_S = \left(1 - \frac{1}{K}\right)\left[\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-2} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-3}\right)\right] > 0, \tag{8}$$

which is strictly positive if and only if the interval of values of c defined by (3) is non empty, always true by Proposition 1. It is also useful to notice that the expression in (8) is equal to the first term of (5), describing the gains which accrue to the seller when at least two indirectly informed buyers happen to like the object, so their bids raise to 1 the price at which the object is won in the auction. We refer so to such term as *rent dissipation* by indirectly informed buyers, since these are rents generated by the information acquisition that the buyer who makes the investment in information will not appropriate, and go instead to the seller of the good.

As argued in the previous section, the term π_{B_N} is strictly positive. Thus the uninformed buyer appropriates some informational rents, by successfully free riding on the information acquisition of all the other buyers, which allows him to get the object at a zero price when nobody else likes it. We indicate then this term as *free riding*. Note that it is exactly equal to the second, negative, term in expression (5) for $\Delta\pi_S$, which reveals that the free riding happens entirely at the expense of the seller of the object and entails so a pure transfer of surplus from the seller to B_N, and hence does not undermine the incentives for efficient information acquisition. What does undermine such incentives, and shows in equation (8), is thus only the rent dissipation.

4 Who should sell information?

Does the inefficiency we found depend on the fact that information is sold by a trader who is also interested in purchasing the object? We examine here the efficiency properties of equilibria when other types of traders can be the providers of information.

4.1 Disinterested traders

As we said in the introduction, a common proposal for solving inefficiencies in information transmission in markets is the separation between information providers and traders. We model this by introducing a new type of agents, who do not own the object nor have any utility for it and hence have no interest in participating in the market where the object is traded. To keep the comparison clean, we assume that also this type of disinterested traders has to pay a cost c to acquire the information.

The reporting strategy of a disinterested trader is clearly different from that of an interested trader since the first one never has an interest in lying over the type of the object. The optimal reporting strategy with maximal degree of truthfulness for the disinterested trader is then to always tell the truth. Hence the quality of the information transmitted is clearly higher and so information can be sold at a higher price. Does this imply the equilibrium with a disinterested trader as provider of information has better efficiency properties? We will show that the answer to such question is negative, the efficiency properties are actually worse in this case, as the incentives for information acquisition are weaker.

Proposition **2** *When information can be sold only by disinterested traders, in equilibrium there is again a less than efficient level of investment in information. Furthermore, the interval of values of c for which information is not acquired in equilibrium though it is* *socially efficient to acquire it is*

$$(N-1)\,\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < c < \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right),$$

which is larger than the one found in Proposition 1 when information is sold by potential buyers.

The intuition for this result is not too hard to get. Notice first that the payoff of a disinterested trader is only given by his revenue from the sale of information, as he never gets any payoff in the auction. Hence information is acquired in equilibrium if the revenue from the sale of information alone exceeds the cost c. As a consequence, there can be an equilibrium where information is sold by a disinterested trader only if he is the monopolist provider of information and information is sold at a strictly positive price (with two or more sellers of information, by the argument given in the previous section, the price of information is always zero).

For the incentives to acquire information to be stronger in the present situation, the higher revenue from the sale of better quality information should more than compensate the lack of any payoff in the auction. The disinterested trader has one additional customer than a potential buyer as he can sell information at a positive price to $N-1$ rather than $N-2$ buyers. Since, as shown in the proof, the price at which information is sold is the same, this means an extra gain from the sale of information equal to $1/K\,[(K-1)/K]^{N-1}$. On the other hand, a disinterested trader does not gain any surplus in the auction, so he loses, with respect to a potential buyer, the surplus this one gets in the auction, which is $(K-1)\,/\,(K)^2$. Clearly, the loss is larger than the gain, which explains the greater region of inefficiency for the disinterested trader.

4.2 The seller of the good

We examine next whether the owner of the good as a provider of information could allow to overcome the inefficiency problem we found. We consider the case where the owner of the good does not ask buyers to report their type. This is in fact the best for him, as we will argue below. If the seller has no information over the buyers' types, he is willing to report truthfully the type of the object, like the uninterested trader; hence the information sold is of the highest quality.

Proposition 3 *When only the owner of the good can be seller of information, in equilibrium there is a less than efficient level of investment in information. In particular, for values of c lying in the following, non empty interval:*

$$\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right) \leq c \leq \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right) \tag{9}$$

information acquisition is socially efficient, but does not take place in equilibrium.

This result can also be easily understood in the light of the discussion in section 3.3. Like in the case where information is sold by a potential buyer of the good, whenever information is acquired equilibrium allocations are always ex-post efficient. Hence, the sum of the changes in the payoff of all traders between the situation with and without information acquisition equals the change in total welfare, $W_1 - W_0$. Since the payoff of buyers who purchase information is again zero in both cases (for c below but close to the threshold for information to be acquired), the change in total welfare equals the change in the payoff of the seller, $\Delta\pi_S^S$, plus the payoff of the buyer (B_N) who remains uninformed.[19] That is[20]:

$$W_1 - W_0 = \pi_{B_N}^S + \Delta\pi_S^S. \tag{10}$$

Thus, underinvestment obtains whenever $\pi_{B_N}^S > 0$. Since the payoff of the uninformed buyer has the same value as when information is sold by a potential buyer, i.e. $\pi_{B_N}^S = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, the result follows. Notice that the source of the inefficiency is here only the informational *free riding* of the uninformed buyer, not the *rent dissipation* by the indirectly informed buyers, in contrast to what we found in section 3.3 when the information provider is a buyer.

Another natural question is whether information acquisition is more efficient if carried out by the owner of the good rather than by a potential buyer. Comparing the threshold for a potential buyer to acquire information derived in theorem 1, c^I, with the one obtained in proposition 3 for the owner of the good, we find that inefficiency is more severe with the owner of the good as information provider when:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} > \left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right),$$

[19]As shown in the proof of Proposition 3 in the Appendix, the seller's optimal choice is always to selle information to all buyers but one.

[20]We use a superscript S to denote variables at equilibria where the owner of the good is the seller of information.

or equivalently,[21]

$$\left(\frac{K-1}{K}\right)^{N-3}\left(\frac{K+N-2}{K}\right) > 1 \tag{11}$$

As shown in Claim 5 in the Appendix, this condition is satisfied if $K-N$ is sufficiently large, or if $K<6$. When N is much smaller than K, the probability that two agents like the same type of object is quite small. Hence, the *rent dissipation*, the source of inefficiency for the buyer as information provider, is small, while the payoff from *free riding* is large with N small.

Should the seller of the good always be trusted as a more reliable source of information than a potential buyer of the good? Before concluding this section we would like to argue that while this is true, as we have seen, in the specific set-up considered earlier, it is not in general and also the owner of the good faces conflicts of interests in his reporting strategy. A first illustration of this fact is provided here, by examining the case where the seller is informed about the buyers' types and establishing the following claim made earlier:[22]

PROPOSITION 4 *Let $c > \frac{1}{K}\left(\frac{K-1}{K}\right)$. If the owner of the good has acquired information and, before sending his report, asks buyers to report their type, in a maximal truth-telling equilibrium he sometimes reports a lie $m \neq v$. In addition, his expected equilibrium payoff is lower than when he does not ask buyers to report their type.*

As shown in the proof in the Appendix, if the owner of the good learns the buyers' type and finds that none of the buyers happen to like the object, he is not willing to report the true type of the object, but prefers to lie and report one of the buyers' types. Similarly, if the seller learns that only one of the buyers likes the object but at least two other buyers like a different type of the object, i.e. say $v=\theta_i$ only for $i=1$ but $\theta_2=\theta_3\neq\theta_1$, he prefers to report θ_2 rather than the truth. The reason is that these lies allow to increase buyers' demand for the object and thus the price at which the object is sold in the auction. In all other circumstances (i.e. when he finds that at least two of the buyers like the object), the owner of the good is willing to report the true type of the object. Thus his report still contains some, though now imperfect, information over the true type of the object.

[21] Note that (11) is the reverse inequality of (6), which describes the condition under which $\Delta\pi_S > 0$. By comparing (7) and (10) we see in fact that a buyer is more efficient than the seller of the good as provider of information, or (11) holds, when $\pi_{B_N^S} > \Delta\pi_S + \pi_{B_N}$; that is, since $\pi_{B_N^S} = \pi_{B_N}$, when $\Delta\pi_S < 0$.

[22] Note that $\frac{1}{K}\left(\frac{K-1}{K}\right)$ is below the threshold for information acquisition found in Proposition 3, $\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right)$. Even though we conjecture the result is also true when the condition $c > \frac{1}{K}\left(\frac{K-1}{K}\right)$ is violated, such condition is the important case to consider to verify whether we can achieve full efficiency with the seller as information provider .

Given the above reporting strategy of the seller, the equilibrium allocation will not always be ex post efficient, as sometimes the object will end up in the hands of a buyer who does not like it even when there is another buyer who likes it[23]. This inefficiency together with the lower informational content of the reports sold adversely affect the buyers' willingness to pay for information and hence the seller's revenue from the sale of information, as well as his revenue from the auction when buyers happen to like the object. We show that altogether such negative effects prevail over the positive effect on the auction revenue which obtains when buyers of information do not like the object, so that the expected payoff of the owner of the good is indeed higher if he does not know the buyers' types.

The conflict of interest we just identified is an important one. Lying about the type of the good in order to increase the demand of the object in the market is akin in fact to the "hyping" of securities by analysts which inspired the counter-measures in title V of the Sarbanes-Oxley act (as well as the authors of the report of the European Commission Forum Group 2003). Such lies happen in spite of the fact that information here is not about the quality, but the variety of the good for sale. It will occur 'a fortiori' when information concerns quality as well, i.e. when elements of vertical differentiation of the information are introduced (see our discussion in section 6).

4.3 Who will sell information?

So far, we considered situations where only one type of trader has the 'license' to sell information. We discuss here briefly what happens when all three types of traders (potential buyers, disinterested traders and owner of the good) can compete among them for the acquisition and sale of information.

The first thing to note is that the entry of a second agent as a seller of information will drive the price of information down to zero, for reasons analogous to those explained in Section 3.2. Given this, the only incentive for an agent to enter the market and disrupt the equilibria with a monopolist provider of information may come from the effects that entering has on the agent's payoff in the auction. This immediately implies that an uninterested trader, whose only payoff is given by the revenue from the sale of information, can never gain from entering and hence never wants to disrupt any monopolist equilibrium. Also, as argued in that same section, when there is already one agent selling information the payoff

[23]This will happen in particular when only one buyer likes the true type of the object while at least two other buyers like the same type: in that event, as we said, the seller prefers to report their type in order to extract a higher price in the auction.

in the auction of a buyer is always the same whether he acquires information directly or indirectly. Since the price at which a monopolist sells information is always less or equal to c, a buyer never strictly gains by entering. This leaves the owner of the good as the only potential disrupter of monopolist equilibria.

When a disinterested trader is a monopolist seller of information, as shown in Proposition 2, he sells information to all buyers except one and always reveals the true value of the object. Hence in that case entry only affects the information available to the single buyer who is not purchasing information and we can verify (see proof of the next Proposition) that the change in the auction revenue of the owner of the object if he enters is zero so that entry is never profitable. But when the monopolist seller of information is a potential buyer, he does not always tell the truth and then entry also affects the information revealed to the agents purchasing information. Consider in particular the case where the cost of acquiring information is low ($c < c^0$). With a potential buyer as an information monopolist the auction price is either 0, as stated in Theorem 1, or 1. If the owner of the good enters the market, the auction price will never be lower and will increase from 0 to 1 in the event where the directly informed buyer likes the good and at least one other buyer also likes it. That event has probability $1/K\left(1-((K-1)/K)^{N-1}\right)$, and one can easily verify that such probability, which equals to the increase in auction revenue, is greater than c^0. Thus there are no equilibria with a buyer as an information monopolist when $c < c^0$. The incentives to enter in the other case ($c \geq c^0$) can be analyzed with a similar reasoning.

More formally, we have:

PROPOSITION **5** *Suppose all types of traders are allowed to acquire information and compete among them for its sale. In this case:*

1. *All the equilibria we found when only the owner of the object or only disinterested traders are allowed to sell information remain (monopolist) equilibria in this case. The same is true also for the (duopoly) equilibria with two potential buyers selling information.*

2. *An equilibrium with a potential buyer as an information monopolist exists only when $c \geq c^0$ and*
$$\frac{1}{K}\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right)\left(1-\frac{1}{K}\right) < (N-2)\frac{1}{K^2}\left(\frac{K-1}{K}\right)^{N-1}+c.$$

Thus we can say that the owner of the object is the most aggressive trader in the pursuit of the sale of information.

5 How can Efficiency be Attained?

We discuss in this section some possible ways to overcome the inefficiencies described in the previous sections. They vary according to the type of the agent selling information.

5.1 Differentiation of the information sold

We allow here informed traders to sell different kinds of reports over their information, at different prices. Thus there can be differentiation of the information transmitted, and the extent of such differentiation will be optimally chosen by the seller. We consider first, in the next subsection, the case where the seller of information is a potential buyer and is always a monopolist, i.e. entry to the market for the sale of information is restricted to a single trader. We then discuss in the following subsection the consequences of eliminating this restriction and allowing, as in the previous sections, for free entry in the market for information.

A (potential buyer as a) monopolist seller

To see why the differentiation of information may allow to increase the profits of the seller of information, recall that, as we saw, when a single type of report is sold the price a buyer is willing to pay for information depends on the number J of other buyers who choose to remain uninformed. Purchasing information is indeed valuable for the buyer because it gives him an informational advantage over the uninformed buyers, which manifests itself in a priority over uninformed buyers in obtaining the good when the buyer likes it. When a single type of report is sold, there are up to three information, and hence priority, levels. First, the directly informed buyer, then all the indirectly informed buyers (who share the same priority level), finally the uninformed buyers, when they exist. The larger is the number J of buyers in the last level, the more valuable is the information of indirectly informed buyers. We now show that, by differentiating the reports sold, the seller of information can arrange the indirectly informed buyers into several distinct priority levels, and by so doing can increase his revenue. Hence, the incentives for information acquisition improve, as the *rent dissipation*, which was shown in Section 3.3 to be at the root of inefficiency in information acquisition, is reduced (if not eliminated).

Notice that, to implement an effective differentiation of the reports sold, the seller must have some information over buyers' preferences. As argued in the previous sections, buyers are only willing to truthfully report their type when the seller is a potential buyer or a disinterested trader but not when he is the owner of the good. Hence the results obtained here hold not only when the monopolist seller of information is a potential buyer but also when he is a disinterested trader, but not when he is the owner of the good.

The seller of information chooses now the number of types of reports and the prices at which he is willing to sell them. We will consider in particular the case where the different types of reports offered for sale can always be arranged in a hierarchy of reports of decreasing quality, or informativeness. Let L denote the number of different types of reports sold. The hierarchy of the qualities of the different reports is modeled by assuming that the seller issues L messages and buyers purchasing a report of type l, $l \in \{1, .., L\}$ observe all the messages m_j, $j = l, .., L$. The information provided by the reports has then a nested structure, in the sense that receiving any report $i > l$ conveys no additional information when compared to report l, while the reverse is not true. Hence report 1 has the highest quality and report L the lowest. We consider again the case where the set of possible messages available to the sellers of information for any l is the set of direct messages, $m_l \in \mathcal{M} = \{1, 2, ..., K\}$. Facing the menu of reports on offer, and the prices p_l, $l = 1, .., L$, at which they sell, each uninformed buyer chooses which report to buy.

We characterize first the equilibria of the subgame starting from the node where a single buyer, say B_1, has acquired information and is selling differentiated information in the market. For any given level of L we determine the optimal choice of B_1 concerning the prices posted for the different reports and the equilibrium strategies in the rest of the subgame (purchase of information by the uninformed buyers B_i, $i = 2, .., N$, reporting strategies and bids in the auction). On this basis we can then find the level of L which maximizes the revenue of the seller B_1. Finally, we compare this value of the revenue to the cost c; when it is higher we conclude that information acquisition is worthwhile for the seller and will take place in equilibrium.

There are still two phases in the reporting of messages. First each buyer of information sends a report over his type to the seller of information. Subsequently the seller sends the messages m_l, $l = 1, .., L$ and, for any $l \in \{1, .., L\}$, the buyers of report of type l receive the messages $(m_l, m_{l+1}, .., m_L)$. We focus our attention on the equilibria where agents' reporting is again characterized by the maximal degree of truthfulness and is now also consistent with the differentiation of information in L levels.

To describe the reporting strategies, it is convenient to adopt some notational conventions. Given the hierarchical structure of the information, we will sometimes refer to the buyers purchasing from B_1 a report of quality l as the buyers in layer l of the hierarchy. For any $l \geq 2$, let $\mathcal{N}_l(B_1)$ denote then the set of buyers purchasing a report of type $i \geq l$ (i.e.

who are in layer l or below) and $N_l(B_1)$ the number of different realizations of θ_i across all buyers $B_i \in \mathcal{N}_l(B_1)$; hence $\mathcal{N}_l(B_1)/\mathcal{N}_{l+1}(B_1)$ indicates the set of buyers in layer l. $\mathcal{N}_1(B_1)$ is similarly defined and indicates the set of buyers who purchased any type of report from B_1. We will show that there is an equilibrium where the uninformed buyers always report their type and the reporting strategy of the seller B_1 for the messages $m_1, .., m_L$ is defined recursively as follows:

$$m_1 = \begin{cases} v, \text{ if } v \neq \theta_1 \\ y, \begin{cases} \text{with probability } \frac{1}{[K-N_1(B_1)]}, \\ \text{for all } y \neq \theta_j, \ \ B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } v = \theta_1 \end{cases} \tag{12}$$

and, for $l = 2, .., L$

$$m_l = \begin{cases} m_{l-1}, \text{ if } \ m_{l-1} \neq \theta_i \text{ for all i} \in \mathcal{N}_{l-1}(\mathrm{B}_1)/\mathcal{N}_l(\mathrm{B}_1) \\ y, \begin{cases} \text{with probability } 1/[K - N_1(B_1)], \\ \text{for all } \ y \neq \theta_j, \ \ B_j \in \mathcal{N}_1(B_1) \end{cases}, \text{ if } \ m_{l-1} = \theta_i \text{ for some } B_i \in \mathcal{N}_{l-1}(\mathrm{B}_1)/\mathcal{N}_l(\mathrm{B}_1) \end{cases} \tag{13}$$

Thus at each layer l the informed trader tells the truth as long as the true variety of the object does not coincide with his own type or with the type of any buyer who has purchased information of higher quality. Otherwise, the informed trader randomizes over any variety different from the type of any of the agents who purchased information.

With this message structure the equilibrium exhibits a few other interesting features.

Proposition **6** *When we allow for the differentiation of the information sold, with a monopolist seller of information there is an equilibrium where information acquisition takes place if and only if*

$$c \leq \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) = c^{I,diff}, \tag{14}$$

the seller adopts the truthful reporting strategy (12), (13) and we have maximal differentiation (a different report is sold to each buyer). Also, information is sold to all other buyers (i.e. $L = N-1$) when

$$c \leq \left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right)\frac{1}{N-2} = c^{0,diff} \tag{15}$$

and otherwise to all other buyers except one ($L = N-2$).

Figure 2 summarizes the result.

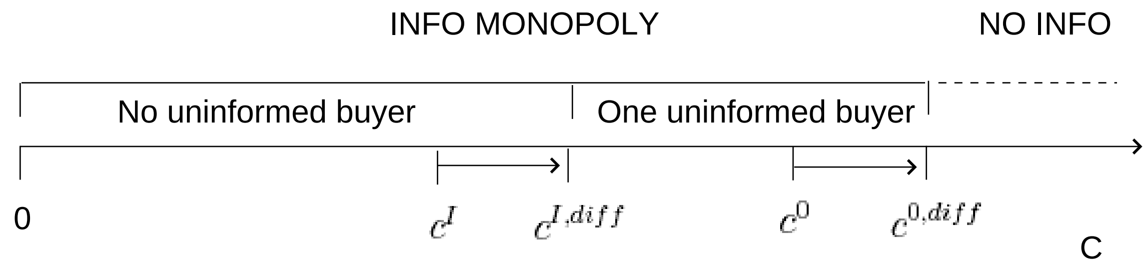

Figure 2: Equilibrium with heterogeneous messages

Thus the optimal choice for a monopolist seller is to design the hierarchy of reports sold and the prices posted so that each buyer chooses to purchase one report, and each type of report is purchased by only one agent. We have so only one buyer of information in each layer, in each priority level, i.e. the maximal degree of differentiation as possible. The key insight in the argument of the proof for this result is that the payoff of a buyer purchasing a given report only depends on the total number of other buyers who are **equally or better informed** than him. Hence the price such buyer is willing to pay for information does not change if anybody who is equally informed gets instead a higher quality report. The one who does that, however, improves his priority level and hence his payoff, and therefore is willing to pay more. Thus when the same type of report is sold to two buyers, the seller of information can always increase his revenue from the sale of information by improving the quality of the report which is sold to one of them. Furthermore, this will have no effect on the seller's payoff in the auction (since the price at which he gets the object only depends on whether or not there are uninformed traders). By induction, the best is to completely differentiate the information sold to each buyer who is purchasing information.

Notice that the threshold for information acquisition to take place with a monopolist seller of differentiated information, $c^{I,diff}$ in (14), is the same as the one found in (2) for the efficiency of the investment in information. Hence the equilibrium is now efficient, not only with regard to the properties of the allocation but also of the information acquisition. This is due to the fact that, by selling a different report to each buyer, the seller of information, B_1, can ensure that all buyers are completely ranked, so that there will never be ties in the auction and hence no *rent dissipation*. Hence B_1 can appropriate now all the social surplus generated by the acquisition of information.[24]

[24]Notice that the *free-riding* by the uninformed buyer still occurs in this case. However, as argued in section 3.3, this only negatively affects the owner of the good, not the seller of information.

Condition (15) reflects a tradeoff for the seller of information that is similar to the one found in the case of homogeneous information. When there is an uninformed buyer the price at which the object is gained in the auction is higher, both for the directly and the indirectly informed buyers, than if everybody purchases information. On the other hand, when there is no uninformed buyer the value of any buyer's outside option, i.e. his payoff if he were to remain uninformed, is higher, so that the maximum willingness to pay and hence the revenue from the sale of information is lower than when not all buyers purchase information. The key difference with respect to the homogeneous information case is that now the provider of information can get a positive revenue from the sale of information also when all buyers purchase a report. Because of this, the maximal level of c at which in equilibrium all buyers purchase information is higher (that is, $c^{0,diff} > c^0$).

Remark **1** *Notice that there is a close relationship between differentiation and resale of information. Our results can in fact be reinterpreted as describing the outcome when the agents who purchase information are able to resell it. In that case in equilibrium a single buyer purchases information from the informed trader, and then sells, at a lower price, a noisier report which is purchased by only one buyer, who in turn sells an even noisier report to a single buyer, and so on.*

Free Entry

In the previous section we have seen that the differentiation of the information sold allows a monopolist to achieve an efficient outcome. However, we show here that it also makes the monopolist's position much more vulnerable to entry. In the case of homogeneous information, we know from Theorem 1 that with free entry in the market for information in equilibrium there is at most a single seller and monopoly is the only outcome for most parameter values. Now this is no longer true.

Proposition **7** *When the cost of information c is not too high, there is always an equilibrium with at least two sellers of information (clearly inefficient). Each of them chooses to sell the same number of different reports as in the monopoly equilibrium and to adopt the same reporting strategies, (12) and (13).*

To understand this result we have to remember what prevented multiple entry when homogeneous information is sold. In that situation a buyer of information was never willing to pay a positive price to purchase an additional report from a second seller. His position was in fact in the intermediate priority level, between the informed sellers of information and the uninformed buyers, and such position was unaffected (i.e. his surplus in the auction unchanged) by the purchase of a second report. Given this, the only possible outcome was intense competition among sellers of information driving its price down to zero.

On the contrary, when the information providers sell differentiated reports every buyer who is considering to buy a report not of the lowest quality needs to buy the same type of report from each seller to be able to retain the position in the hierarchy of information - and hence the priority level - such report is meant to deliver. If he fails to buy this report from one of the providers, the traders who buy reports of lower quality from such provider would be able to compete with him for the object on the same terms, hence the buyer would not get the object with a positive payoff when both he and any of these other traders like the object. This fact creates the possibility of a collusive outcome where each seller of information, instead of lowering prices so as to attract all customers only to him, prefers to set prices in such a way that every buyer will purchase a report from all the sellers. In this way the seller can gain positive profits by sharing the monopoly rents with the other sellers.

The outcome with a monopolist seller of heterogeneous information can also be sustained at an equilibrium with free entry. It can in fact be easily verified that the strategies where, whenever another trader enters the market for the sale of information, each seller chooses not to differentiate the information, i.e. to sell a homogeneous report, constitute another equilibrium of the subgame. In such situation, as shown in Section 3, information is sold at a zero price and hence entry is not profitable. However this requires the sellers of information to coordinate on what is, from their point of view, a Pareto inferior subgame equilibrium.

5.2 Entry fee for the auction

We propose here an alternative way to restore first best efficiency, when the seller of information is the owner of the good. Suppose he could charge any potential buyer an entry fee to participate in the auction. Then if the owner of the good sets such fee equal to the rent obtained by the uninformed individual in the equilibrium without entry fee (which is, remember, $(K-1)^{N-1}/K^N$) and rebates it to all those traders who purchase information from him, he can extract from all the buyers the full surplus generated by the information. Hence the social incentives for information acquisition are aligned with the individual incentives for the owner of the good and efficiency obtains. At the same time, we should point out that this is only true if the owner of the object has no information over the buyers' types and can credibly commit not to seek such information. When these conditions are not met, as we saw in Section 4.2, the outcome is always inefficient when the seller of information is the owner of the good.

6 Discussion

The model considered is quite versatile and has allowed us to study information acquisition, transmission and trade upon this information in a number of setups. The information transmitted may be homogeneous or heterogeneous, the seller of information may be a potential buyer of the good, the seller or a "neutral" third party. We show next that the analysis and conclusions drawn are fairly robust with respect to changes in various features of the specification. Finally we discuss the consequences of introducing some elements of vertical differentiation in the uncertainty concerning the object traded for the efficiency of the market and for the conclusions drawn concerning the desirability of different forms of regulatory interventions.

6.1 Some robustness checks

6.1.1 Organization of the market for information

How would the equilibrium properties change with alternative assumptions concerning who and when the price for information is posted? Suppose the sellers of information were to post the price after - rather than before - having learnt the signal realization. Then the price posted would have a signaling value, as a seller may want to post a different price after having learnt that he likes or does not like the object. In this case we should expect a large set of equilibria to exist. Clearly an equilibrium with properties analogous to the ones found in Theorem 1 (where the price posted conveys no information) still exists. There are other pooling equilibria, but with lower prices (and thus lower efficiency),[25] and under maximal truth-telling there is no separating equilibrium.[26]

Consider next the case where the price is posted by buyers, rather than sellers of information, again before the latter have learnt the realization of the signal. Each buyer can then

[25] Those are sustained by the off-equilibrium beliefs that a deviator posting a higher price is a buyer who likes the object, from whom nobody wants to purchase information.

[26] At a separating equilibrium there are two distinct prices which signal that a seller of information likes, respectively does not like, the object. At the first one the seller is expected to bid 1 and so no buyer is willing to pay anything for information. At the second one the seller is expected to tell the truth (under maximal truth-telling), thus information is valuable and its sale also allows to decrease the price at which the object is gained in the auction. Thus, the seller of information would strictly gain by deviating when he likes the good and posting the second price.

post a price contingent on the number of other buyers who also purchase information[27]. We claim that in such case the equilibria would have similar, though not exactly identical, features to the ones we found. In particular, each buyer of information sets a price equal to his maximal willingness to pay for the information, except when the number of the other buyers of information equals its maximum minus one (e.g. $N-3$ with a monopolist seller given by a potential buyer), in which case he sets a price equal to zero, which is lower than his true maximal willingness to pay for information. This is because, if all buyers of information were to report their true willingness to pay also when the number of other buyers is $N-3$, and the seller were indeed to sell to a total of $N-2$ buyers, i.e. to all potential buyers but one, each buyer of information would have an incentive to deviate and post a lower price. By so doing the buyer would make sure he is the one not purchasing information, i.e. remaining uninformed, which gives him a strictly positive payoff. Given these strategies of the buyers, the optimal response of the seller of information is, for c low to sell information to everybody at a zero price (as in region 3. of Theorem 1) and, for intermediate values of c, to sell at a positive price to all the potential buyers less two (unlike in region 2. of the Theorem). In this second case the payoff of all potential buyers of information, both those who actually purchase it and those who don't, is zero.

What are then the consequences of alternative specification of the message subgame? Suppose the set $\mathcal{M}$ of available messages is enlarged to include also an empty message, a blank report. This message allows an alternative form of deception of buyers: whenever the seller of information is interested in the object and thus faces a conflict of interest, he could send buyers a completely uninformative message, i.e. one that does not change their priors (which can be interpreted as providing buyers with no advice, or report). It is fairly easy to verify that in all the equilibria where this alternative form of deception is used the utility of the seller of information is lower than in the one considered before (with reporting strategy (1))[28] and hence the underinvestment in information problem more severe. On the other hand, if phase a. of the message game did not exist, so that only the seller of information can send a report to buyers, the seller would no longer be able to deceive buyers by telling them, in some at least partially credible way, they are not interested in the object. The best form of deception for the seller, when the set of available messages is $\mathcal{M}$ enlarged to include an empty report, is indeed to send such report, i.e. a completely uninformative message. Hence his payoff would be lower than when phase a. exists or the seller (if he is

[27]It is easy to verify such property is needed for the equilibrium not to be trivial.

[28]The key reason is that, on receipt of this completely uninformative message the optimal bid of buyers is $1/K$, not 0.

a potential buyer) has some information about buyers' preferences. One could argue that in many situations the providers of information indeed have such information and use it in their reports.

6.1.2 Auction Format

We have assumed throughout that the object is sold via a second price auction. Since information transmission generates correlation in the values of the agents, one cannot invoke revenue equivalence to justify such choice. However, we can argue at least that a first price auction is likely to generate less rents for the informed buyer and possibly ex-post misallocations (hence worse incentives for information acquisition and lower welfare). The reason is that with a first price auction a pure strategy equilibrium does not exist. To fix ideas, think of a message strategy as in (1). Any (pure strategy) bid by the informed buyer (when the good is of his preferred type) *above* the expected value of the other buyers (after receiving the report sent by the informed buyer) would induce them to bid zero , as that would be their valuation conditional on winning the auction. This would make the bid of the informed buyer suboptimal. But any (pure strategy) bid by the informed buyer *below* this expected value would induce the other buyers to bid above the informed buyer, which would not be optimal for him either. Pure strategy bids by indirectly informed or uninformed buyers could not be equilibrium strategies for analogous reasons. A mixed strategy equilibrium clearly leads to ex-post suboptimal allocations with positive probability, higher prices in the auction for the informed buyer and lower revenues from selling information.

Another assumption is that a single unit of the object is available for sale. Suppose multiple - say $Q > 1$ - units of the good were up for sale and each buyer has a positive utility only for one unit, thus a limited capacity for "enjoying" the good in the market. Let the good be sold via a $(Q+1)$-th price auction. When the seller of information likes the object, if there is no differentiation of the quality of the information sold, he will lie to all buyers so as to lower competition in the auction, as he does when there is a single unit. But now this will lead to some units of the object being sold to buyers who do not like them and hence to a possible ex-post inefficiency of the equilibrium allocation. Notice that this problem does not arise if the seller differentiates the quality of the information sold, as in Section 5.1; with such differentiation is allowed, the efficiency of the equilibrium could be preserved.

6.2 Introducing Vertical Differentiation of the Information

In the environment considered so far the uncertainty only concerns a horizontal differentiation element, the type or variety of the object over which buyers have idiosyncratic tastes. It is thus important to examine the consequences of allowing also for the presence of a vertical differentiation element, quality, over which buyers' preferences tend to agree.

To this end, consider the following extension of the model. Suppose the good not only comes in one of the K types we described, but also in one of 2 quality levels, H (High) and L (Low). Formally, the true type of the good is now $v = (k, q) \in \mathcal{S} \equiv \mathcal{K} \times \{H, L\}$. Suppose, in addition, that buyers are also of two types: while all buyers only care for one, randomly drawn variety, some of them are sensitive to quality (Se) and others are insensitive to quality (In). An In consumer has a constant valuation of 1 for a good of the variety he likes. A Se consumer values a good of the type he likes V if the good is of H quality, and 0 if it is of L quality. Let us assume for simplicity that H and L have identical probabilities for each variety of good, and that consumers have identical probabilities to be of type Se and In.

Also, consider again the case where the set of available messages to the seller of information is the set of direct messages: $\mathcal{M}' = \mathcal{S}$, so that a generic message m is now a pair (k, q), where $k \in \mathcal{K}$, $q \in \{H, L\}$. We show next that, when the seller of information is the owner of the object, whenever he perfectly reveals in equilibrium the true variety k of the object, he never reveals any information over the quality of the object:

Proposition 8 *Suppose the set of possible types of the good is given by $\mathcal{S}$ and the owner of the object is the seller of information. Then at any equilibrium where the seller truthfully reveals the variety of the object, that is where for all $m \in \mathcal{M}'$, $\Pr(k = i|m) = 1$ for some $i \in \mathcal{K}$, we have $\Pr(q = H|m) = \Pr(q = H) = \frac{1}{2}$. Thus, if $V > 2$, the Se type buyers always bid more for the good than the In type buyers, and vice versa if $V < 2$.*

Hence we can never have at the same time perfect revelation of the true variety and the true quality of the object.[29] It follows that the equilibrium allocation of the good is ex post inefficient, which in turn implies that not all social surplus can be appropriated by the seller, and so that there will be underinvestment in information acquisition.

The source of the inefficiency is that when the seller of information is the owner of the object he faces a conflict of interest, analogous to the one we found in the second part of Section 4.2, which induces him to want to lie to exaggerate how much buyers like the good

[29] Some revelation over the true quality of the object could be obtained at equilibrium, but only at the cost of partial revelation over the true variety.

so as to increase his revenue from the sale of the good. Notice that analogous features - ex post inefficiency due to false reports - obtain when the seller of information is a potential buyer (the circumstances and form of the lie are however different in that case: the seller wishes to lie only when he likes the object and tells buyers the variety and quality of the good is not "right for them"). The only case where the report sent is always truthful and the equilibrium allocation remains ex post efficient is the one where the seller of information is a disinterested trader.

7 Conclusion

Good quality information is key to a properly functioning market. This has long been understood by academics as well as by practitioners and regulators. But market participants are *not endowed* always with all necessary information, and typically *obtain* it, often from other actors in the market. For this reason, authorities have established numerous rules on the amount and kinds of communication between market participants and information providers. Surprisingly, there is little research into the interplay between acquisition of information, its transfer and actions in the market, which would be necessary to provide foundations for such policy. We partly fill this gap by building a formal model of a market environment with costly acquisition and unverifiable transmission of information. In this set-up we can investigate the conflicts of interest faced by the information providers, see how they vary according to the type of the provider, in which directions they limit the extent of truthful transmission of information and examine the consequences for the performance of the market.

We find that when information concerns a prevalent horizontal differentiation component, there are typically inefficiencies because of underinvestment in information acquisition. Usual regulatory interventions, such as firewalls, or limiting the sale of information to parties which have no interest in trading the underlying object, worsen the inefficiencies. In contrast, efficiency can be attained with a monopolist selling differentiated information, if additional entry is blocked. When, on the other hand, the vertical differentiation element is more relevant, firewalls can be beneficial. As we argued in the introduction, both the horizontal and the vertical elements are likely to be part of the information problem in real markets. We thus provide a tool to assess the potential benefits of establishing various kinds of regulations.

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Appendix

Proof of Theorem 1 (further details)

We provide here the missing details of the proof of the Theorem.

Optimal pricing rules and payoffs with a monopolist seller of information

As argued in Section 3.2, to find the optimal pricing strategy of a monopolist seller of information in the subgame starting in stage 2 of the game, we derive first the maximal willingness to pay for information of an uninformed buyer for each given number J of buyers who choose not to acquire information. Let us denote such situation as configuration J, where $J \in \{0, 1, .., N-2\}$.

Let, w.l.o.g., B_1 be the seller of information, $B_2, .., B_{N-J}$ indicate the traders buying information from the single seller and $B_{N-J+1}, ..., B_N$ be the J buyers not purchasing information in configuration J, i.e. when there are $J \geq 0$ buyers not purchasing information. The payoff of buyer B_i in such configuration is then denoted by $\pi^J_{B_i}$. The value of the outside option for the buyers purchasing information is given by $\max\left\{\pi^J_{IC}, \pi^J_U\right\}$, where π^J_{IC} (resp. π^J_U) indicate the expected utility of buyer $B_2, .., B_{N-J}$ if, rather than purchasing information, he were to acquire information directly (resp. to stay uninformed). For the buyers not purchasing information it is given by $\max\left\{\pi^J_{UC}, \pi^J_I\right\}$, where π^J_{UC} (resp. π^J_I) is now the expected utility of a buyer $B_{N-J+1}, ..., B_N$ if, rather than staying uninformed, he were to acquire information directly (resp. to purchase information). Let then $p(J)$ be the price posted by the seller of information, that is the price which maximizes his revenue among all the prices that support such configuration.

$J > 0$. From the discussion in Section 3.2, it is immediate to see that when $J > 0$ the payoffs of the N potential buyers are:

$$\begin{aligned}
\pi^J_{B_1} &= \frac{1}{K}\left(1 - \frac{1}{K}\right) + (N - (J+1))p(J) - c \qquad (16)\\
\pi^J_{B_i} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1 - \frac{1}{K}\right) - p(J), \text{ for } i = 2, ..., N-J\\
\pi^J_{B_N} &= \begin{cases} ... = \pi^J_{B_{N-J+1}} = 0 \text{ if } J \geq 2 \\ \left(\frac{K-1}{K}\right)^{N-1} \frac{1}{K} \text{ if } J = 1 \end{cases}
\end{aligned}$$

The value of the outside options for those who are also buyers of information is then

$$\begin{aligned}
\pi^J_{IC} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)}\left(1 - \frac{1}{K}\right) - c\\
\pi^J_U &= 0
\end{aligned}$$

while for the uninformed buyers it is

$$\begin{aligned}\pi_{UC}^{J} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right)-c\\ \pi_{I}^{J} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\left(1-\frac{1}{K}\right)-p(J)\end{aligned}$$

if $J \geq 2$ and

$$\begin{aligned}\pi_{UC}^{1} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-c\\ \pi_{I}^{1} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-p(1)\end{aligned}$$

if $J = 1$.

It is then immediate to see that the optimal pricing rule of the monopolist seller of information in any configuration $J \geq 1$ is to set the price equal to the maximal willingness to pay for information of traders $B_2, .., B_{N-J}$, i.e.:

$$p(J) = \min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}\right\}. \tag{17}$$

Furthermore, configuration $J = 1$ is always attainable since from the above expressions we see that the condition $\pi_{B_N}^{1} = \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} \geq \max\{\pi_{UC}^{1}, \pi_{I}^{1}\}$ holds with the pricing rule in (17), while any other configuration $J > 1$ is only attainable as long as

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J+1}. \tag{18}$$

When (18) holds, $\pi_{B_{N-J+1}} = ... = \pi_{B_N} = 0 \geq \max\left\{\pi_{UC}^{J}, \pi_{I}^{J}\right\}$ is also satisfied, when it is violated no configuration $J > 1$ is attainable since the uninformed buyers always prefer, no matter what is the level of p, to become directly informed.

J = 0. In the configuration $J = 0$ (no one stays uninformed) we have instead:

$$\begin{aligned}\pi_{B_1}^{0} &= \frac{1}{K}+(N-1)p(0)-c\\ \pi_{B_i}^{0} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-p(0) \text{ for } i = 2, .., N\end{aligned}$$

and the value of the outside options for the buyers of information is

$$\begin{aligned}\pi_{IC}^{0} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-c\\ \pi_{U}^{0} &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\end{aligned}$$

Thus $\pi_U^0 > \pi_C^0$ and the optimal pricing rule for such configuration is

$$p(0) = 0,$$

again equal to the maximal willingness to pay of buyers.

On this basis we can provide now the:

Proof of Claim 1. Suppose first that

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^2,$$

so that all configurations are sustainable, since (18) holds for all $J > 1$, and $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for all $J = 1,..,N-2$. We then show that the revenue from the sale of information is always higher in configuration J than in $J+1$, for all $J = 1,..,N-3$:

$$\begin{aligned} (N-(J+1))p(J) \quad &\geq \quad (N-(J+2))p(J+1) \\ &\iff (N-(J+1))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J} \geq (N-(J+2))\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-(J+1)} \\ &\iff \frac{(N-(J+1))}{(N-(J+2))} \geq \frac{K}{K-1} \\ &\iff K-1 \geq N-(J+2), \end{aligned}$$

always satisfied since $K > N$.

Consider next the case where

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}+1} \leq c < \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-\bar{J}} \tag{19}$$

for some $\bar{J} \in \{2,..,N-2\}$, so that only configurations $J = 1,..,\bar{J}$ are sustainable, $p(J) = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$ for $J = 1,..,\bar{J}-1$ and $p(\bar{J}) = c$. By the same argument as above, the revenue is higher at $J = 1$ than at any other $J = 2,..,\bar{J}-1$. Thus we only need to compare the revenue in configuration $J = 1$ with the one at $J = \bar{J}$ (where $p(\bar{J}) = c$) and show that, for all $\bar{J} \in \{2,..,N-2\}$:

$$\begin{aligned} (N-2)p(1) \quad &\geq \quad \big(N-(\bar{J}+1)\big)p(\bar{J}) \\ &\iff (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-(\bar{J}+1))c \end{aligned}$$

Clearly it suffices to show this property for the minimum value of $\bar{J}$, $\bar{J} = 2:$

$$(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-3)c$$

Using the same argument as in the previous paragraph we can show that the following inequality, which is stronger by (19), holds:

$$(N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq (N-3)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}.$$

Since configuration $J=1$ is always attainable and gives also a higher revenue than $J=0$ ($p(1)$ as in (17) is strictly positive while $p(0)=0$). ■

Proof of Claim 2. Given Claim 1, to find the optimal pricing rule of the monopolist seller of information it suffices to compare the choice of $p(1)$ with that of $p(0)$, i.e. to find when

$$\pi_{B_1}^1 \geq \pi_{B_1}^0 \tag{20}$$

holds. Using the expressions derived above for such payoffs and the prices, we get:

$$\pi_{B_1}^1 = \frac{1}{K}\left(1-\frac{1}{K}\right) + (N-2)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\} - c \geq \pi_{B_1}^0 = \frac{1}{K} - c$$

When $c \leq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, the above condition reduces to:

$$c \geq \frac{1}{(N-2)K^2},$$

while when $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$ it becomes

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \geq \frac{1}{N-2}\frac{1}{K^2}$$

Combining these two inequalities we obtain so the property we wanted to prove:

$$\pi_{B_1}^1 \geq \pi_{B_1}^0 \Longleftrightarrow c \geq \frac{1}{N-2}\frac{1}{K^2} = c^0 \tag{21}$$

■

Equilibrium pricing rules and payoffs with an information oligopoly An alternative situation which may arise in the subgame starting in stage 2 has $M \geq 2$ sellers of information (w.l.o.g. let them be $B_1, ..., B_M$) and $N-M$ buyers of information ($B_{J+1}, ..., B_N$). Let us denote it as configuration $OL(M)$. In this case, as we argued in the main text, each seller posts a zero price for information and each uninformed buyer purchases information from all sellers. The payoffs in this case are:

$$\pi_{B_1}^{OL(M)} = ... = \pi_{B_M}^{OL} = \tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-1} - c$$
$$\pi_{B_i}^{OL(M)} = \tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-1}, \ i = M+1, .., N$$

Payoffs with no sale of information The last possibility is configuration No, where no buyer acquires information, hence all buyers stay uninformed and make a bid equal to their expected valuation, $1/K$. The object is then randomly allocated to one buyer, who pays for it an amount equal to his expected value for the good and hence gets no surplus and the payoff of every buyer is:

$$\pi_{B_i}^{No} = 0 \text{ for all } i = 1, .., N$$

Proof of Claim 3. From Claims 1 and 2 it follows that no information is gathered in equilibrium when:

$$\pi_{B_i}^{No} = 0 \geq \begin{cases} \pi_{B_1}^1 = , \text{ if } c \geq c^0 \\ \pi_{B_1}^0, \text{ if } c < c^0 \end{cases} .$$

The second set of conditions, $0 \geq \pi_{B_1}^0$ and $c < c^0$, can be equivalently written as:

$$\frac{1}{N-2}\frac{1}{K^2} > c \geq \frac{1}{K},$$

which never holds. The first one, $0 \geq \pi_{B_1}^1$ and $c \geq c^0$, can be rewritten as

$$\begin{aligned} c &\geq \max\left\{\frac{1}{N-2}\frac{1}{K^2}, \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\min\left[c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right]\right\} \\ &= \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\min\left[c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right]. \end{aligned}$$

Such condition may only be satisfied if $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$, in which case it reduces to:

$$c \geq \frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} = c^I. \tag{22}$$

■

Proof of Claim 4. To get an information duopoly at an equilibrium of the overall game we need:

$$\pi_{B_2}^{OL(2)} \geq \begin{cases} \pi_{B_i}^1, \text{ for } i = 2, .., N-1, \text{ if } c \geq c^0 \\ \pi_{B_i}^0, \text{ for } i = 2, .., N, \text{ if } c < c^0 \end{cases} \tag{23}$$

$$\pi_{B_i}^{OL(2)} \geq \pi_{B_3}^{OL(3)}, \text{ for } i = 3, .., N \tag{24}$$

When $c \geq c^0$, condition (23) can be rewritten as

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - p(1) = \max\left\{\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c, 0\right\}.$$

which holds if and only if $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \geq 0$. On the other hand, if $c < c^0$ (23) becomes:

$$\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \geq \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1},$$

which is never satisfied. Finally, it is immediate to see that condition (24) always holds. We conclude that a duopoly obtains as an equilibrium outcome of the overall game when $c^D \geq c \geq c^0$.■

This completes the analysis of the possible configurations which may arise at equilibrium and hence the proof of Theorem 1. ■

Proof of Proposition 1

The result follows immediately by comparing the threshold for efficient information acquisition, found in (2), with the threshold found in Theorem 1 for information acquisition not to take place in equilibrium, given by (22). We show next that the interval of values of c identified in condition (3) is non empty, i.e.:

$$\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} < \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right).$$

It is easy to verify that this inequality is equivalent to:

$$\left(1 - \frac{1}{K}\right)^{N-3}\left(1 + \frac{N-3}{K}\right) < 1. \tag{25}$$

Since the term on the left hand side approaches one as $K \to \infty$, it suffices to show that this term is always strictly increasing in K to be able to conclude that (25) holds for all K, N. Notice that a term is increasing if its logarithm is increasing. Taking then the logarithm of the left hand side of (25) and differentiating it with respect to K yields:

$$\frac{(N-3)}{K^2}\left(\frac{1}{\left(1-\frac{1}{K}\right)} - \frac{1}{\left(1+\frac{N-3}{K}\right)}\right) = \frac{(N-3)}{K^2}\left(\frac{\frac{N-2}{K}}{\left(1-\frac{1}{K}\right)\left(1+\frac{N-3}{K}\right)}\right),$$

which is strictly positive since we always have $K > 1$ and $N > 3$. ■

Proof of Proposition 2

The maximal price a buyer is willing to pay for information when a total number J of buyers stay uninformed is obtained as in the proof of Theorem 1 and again given by $\min\left\{c, (K-1)^{N-J}/K^{N-J+1}\right\}$. By a similar argument we can also show that a result analogous to Claim 1 still holds when information is sold by an agent different from a potential buyer: the revenue from the sale of information of a monopolist seller is maximal when information is sold to all buyers except one, i.e. in this case to $N-1$ buyers. Hence, if information is acquired and sold by a disinterested trader his payoff, equal to this revenue, is:

$$\pi_{Dis} = (N-1)\min\left\{c, \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\right\} - c \geq 0. \tag{26}$$

It is immediate to see from (26) that an equilibrium exists with information acquisition by a disinterested trader if:

$$c \leq (N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}.$$

Hence to prove the result it suffices to show that the threshold found in (22) for information acquisition to take place at an equilibrium when information is sold by a potential buyer is higher:

$$\begin{aligned}\frac{1}{K}\left(\frac{K-1}{K}\right) + (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} &> (N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \\ &\iff \frac{1}{K}\left(\frac{K-1}{K}\right) > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\end{aligned}$$

which is clearly always true. ■

Proof of Proposition 3

The payoff of the seller of the good when he acquires and sells information is given by the sum of his information and auction revenues. As already argued in the proof of Proposition 2, the information revenue is maximal when information is sold to all buyers except one. The auction revenue, when the number of buyers remaining uninformed is $J \geq 2$, is either $1/K$ (if no informed buyer likes the object) or 1 (if more than one informed buyer likes the object). Thus, when $J \geq 2$ the auction revenue decreases with J, as the probability that more than one informed buyer likes the object is decreasing in J. Putting this together with the fact that the revenue from the sale of information is also decreasing in J for $J > 0$, we get that the payoff of the seller of the object is higher in configuration $J = 2$ than in any other configuration $J > 2$. To find the optimal choice of the seller it remains so to compare his payoff at $J = 2$ with that at configurations $J = 0$ and $J = 1$.

The payoff when $J = 2$ is[30]

$$\begin{gathered}(N-2)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-2} + \left(1 - \left(\tfrac{K-1}{K}\right)^{N-2} - (N-2)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-3}\right) + \\ \tfrac{1}{K}\left(\left(\tfrac{K-1}{K}\right)^{N-2} + (N-2)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-3}\right) - c = 1 - \left(\tfrac{K-1}{K}\right)^{N-1} - c.\end{gathered}$$

[30] This expression holds when the cost c is not too low (the case of interest when investigating the threshold for information acquisition), or $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}$, in which case information is sold at a price $\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}$.

When $J=1$ the seller's revenue from the auction is either 0 (if no buyer who purchases information likes the object), $1/K$ (if exactly one buyer who purchases information likes the object), and 1 otherwise. Summing the information revenue the seller's payoff is:

$$(N-1)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-1}+\left(1-\left(\tfrac{K-1}{K}\right)^{N-1}-(N-1)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-2}\right)+ \\ +\tfrac{1}{K}\left((N-1)\tfrac{1}{K}\left(\tfrac{K-1}{K}\right)^{N-2}\right)-c=1-\left(\tfrac{K-1}{K}\right)^{N-1}-c,$$

identical to the payoff when $J=2$. When $J=0$, since the price at which information is sold is zero, the seller's payoff is simply his auction revenue (equal to 0 in this case if no buyer, or exactly one, likes the object, and 1 otherwise):

$$1-\left(\frac{K-1}{K}\right)^{N}-N\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}-c$$

It is then immediate to see that

$$1-\left(\frac{K-1}{K}\right)^{N-1}>1-\left(\frac{K-1}{K}\right)^{N}-\frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1}\iff\frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1}>\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}$$

always holds. Thus, the seller's payoff is maximized by setting the price of information so that $J=1$ or $J=2$. Information is acquired in equilibrium if the seller's payoff with $J=1$ (or 2) exceeds his payoff without information (which for the seller is not 0 but $1/K$):

$$1-\left(\frac{K-1}{K}\right)^{N-1}-c\geq\frac{1}{K}. \tag{27}$$

The claim of the Proposition follows by comparing the threshold for efficient information acquisition, found in (2), with the one implicitly defined by (27) and verifying the latter is strictly smaller:

$$\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-2}\right)<\left(\frac{K-1}{K}\right)\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right),$$

always true. ■

Claim 5 *When $K-N$ is sufficiently large, or $K<6$, efficiency is higher if information is provided by a potential buyer rather than by the owner of the object (condition (11) holds). Otherwise, the opposite holds.*

Proof. Since the term on the left hand side of (11) approaches one as $K\to\infty$, if for K large such term is strictly decreasing in K we can say that condition (11) holds, for

K large. Taking the logarithm of the term on the left hand side of (11) and differentiating it with respect to K we get:

$$(N-3)\frac{1}{K^2}\frac{1}{\left(1-\frac{1}{K}\right)}-\frac{(N-2)}{K^2}\frac{1}{\left(1+\frac{N-2}{K}\right)}=\frac{1}{K^2}\left(\frac{\frac{(N-2)^2-K}{K}}{\left(1-\frac{1}{K}\right)\left(1+\frac{N-2}{K}\right)}\right)$$

which is strictly negative if and only if $(N-2)^2 < K$. This implies that the term on the left hand side of (11) is first increasing and then decreasing in K. Hence condition (11) necessarily holds for K sufficiently large (relative to N). Furthermore, it holds for all $K > N$ if (11) is satisfied for $N = K$, or:

$$2\left(\frac{K-1}{K}\right)^{K-2} > 1,$$

which is true for $K < 6$ and false for $K \geq 6$. ■

Proof of Proposition 4

Denote by P the price paid to gain the object in the auction and by p the price of information. Let then B_i^{Win} be the event in which buyer B_i wins the auction and $\mathbb{E}_{B_i}$ the expectation conditional on B_i's information at stage 2 of the game. The general expression of the payoff of a buyer of information is:

$$\pi_{B_i} = \Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i) - \mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win}) - p,$$

where the probabilities clearly depend on the number $J > 0$ of buyers who remain uninformed, on who sells information and his reporting strategy. When there is a monopolist seller of information its price is then:

$$p = \Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i) - \mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win}) - \max\{\pi_{IC}^J, \pi_U^J\}, \tag{28}$$

where π_{IC}^J and π_U^J are as in the proof of Theorem 1.

Using (28), the equilibrium payoff of the seller of the good when he is also the monopolist seller of information and there are $J > 0$ uninformed buyers (say B_i, $i = N-J+1, .., N$), both when he knows and does not know buyers' types, is given by the following expression:

$$\begin{aligned}\pi_S^{S,J} \;&=\; \mathbb{E}_S(P) + \sum_{i=1}^{N-J}\left(\mathbb{E}_S\left(\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i)\right) - \mathbb{E}_S\left(\mathbb{E}_{B_i}(P|B_i^{Win})\Pr(B_i^{Win})\right)\right)\\ &-(N-J)\max\{\pi_C^J(S), \pi_U^J(S)\} - c = \sum_{i=N-J+1}^{N}\mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \qquad (29)\\ &+\sum_{i=1}^{N-J}\mathbb{E}_S\left(\Pr(B_i^{Win}|v=\theta_i)\Pr(v=\theta_i)\right) - (N-J)\max\{\pi_C^J(S), \pi_U^J(S)\} - c,\end{aligned}$$

where $\mathbb{E}_S$ is the expectation conditional on the seller's information. When $c > \frac{1}{K}\left(\frac{K-1}{K}\right)^{N-J}$, if the seller has no information on buyers' types, as argued in the proof of Proposition 3, $\max\{\pi_C^J(S), \pi_U^J(S)\} = 0$ and the equilibrium payoff of buyers of information is zero, for any $J \geq 1$. Also, whenever there is at least one (indirectly) informed buyer who likes the object one of them always gets it, hence the surplus extracted by the seller of the object in that event equals the total surplus $\Pr(v = \theta_i$, for some $i = 1,..,N-J)$ and (29) reduces to:

$$\begin{aligned}\pi_S^{S,J,no\inf o} &= \sum_{i=N-J+1}^{N} \mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win}) \\ &\quad + \Pr(v = \theta_i,\text{ for some } i = 1,..,N-J) - c\end{aligned} \tag{30}$$

Whenever $J \geq 2$, since $\mathbb{E}_S\left(P|B_i^{Win}\right) = 1/K = \Pr(\theta_i = v)$ for any $i > N-J$, the term $\sum_{i=N-J+1}^{N}\mathbb{E}_S\left(P|B_i^{Win}\right)\Pr(B_i^{Win})$ has the same value whether or not the seller of information has any information over buyers' types. Since, as argued above, the maximal value of $\sum_{i=1}^{N-J}\mathbb{E}_S\left(\Pr(B_i^{Win}|v = \theta_i)\Pr(v = \theta_i)\right) - (N-J)\max\{\pi_C^J(S), \pi_U^J(S)\}$ is $\Pr(v = \theta_i$, for some $i = 1,..,N-J)$ it follows that $\pi_S^{S,J,no\inf o} \geq \pi_S^{S,J,\inf o}$.

We show next that the same is true when $J = 1$. In particular, we will show that, when the seller ask buyers to report their type, in equilibrium the uninformed buyer never gets the object, $\Pr(B_N^{Win}) = 0$. Hence the first term of (29) is zero. Recalling that, as argued in the previous paragraph, the other two terms of (29) are maximal at the equilibrium where the seller has no information over buyers' types, the result follows.

The message subgame is an in Section 2. At an equilibrium with maximal degree of truthfulness of agents reporting the seller adopts the following reporting strategy:

$i)$ if there is at least one pair $i, j \in (1,..N-1)$ such that $\theta_i = \theta_j$ (i.e. there is a 'tie') :

$$m_S = \begin{cases} v \text{ if } v = \theta_i = \theta_j \text{ for some } i, j \in (1,..N-1), \\ \theta_i \neq v \text{ if } v = \theta_k \text{ for at most one } k \in (1,..,N-1) \end{cases}$$

$ii)$ if $\theta_i \neq \theta_j$ for all $i, j \in (1,..,N-1)$ (there are no 'ties'):

$$m_S = \begin{cases} v \text{ if } v = \theta_i \text{ for some } i \in (1,..N-1), \\ \theta_i \neq v \text{ with probability } 1/(N-1) \text{ if } v \neq \theta_i \text{ for all } i = 1,..,N-1 \end{cases} \tag{31}$$

Thus the seller tells the truth only when at least two buyers of information like the object (there is a tie on the truth) or all buyers like a different type (there are no ties) and one of them likes the object. On the other hand buyers still report truthfully their type:[31]

[31] The argument is similar to Theorem 1. By lying, the informational content of a 'good' report, equal

CLAIM **6** *For any buyer who purchases information it is optimal to truthfully report his type to the seller (given that all other buyers do that and the seller follows the reporting strategy in (31).*

To complete the argument for the case $J = 1$ it remains then to verify that

CLAIM **7** *Given the seller's reporting strategy in (31), there is always at least one indirectly informed buyer whose bid is strictly greater than* $1/K$, *and thus* $\Pr(B_N^{Win}) = 0$.

We are then left with establishing the result for $J = 0$. In such case, from (29) we get

$$\pi_S^{S,0,\inf o} \leq \sum_{i=1}^{N} \mathbb{E}_S \left(\Pr(B_i^{Win} | v = \theta_i) \Pr(v = \theta_i) \right) - c.$$

The equilibrium reporting strategy of the seller is analogous to the one described in equation (31), and buyers still want to report truthfully their type[32]. Thus the only possible misallocation of the object is when only one of the N buyers likes the object and there is a tie on a different type, so that

$$\begin{gathered} \textstyle\sum_{i=1}^{N} \mathbb{E}_S \left(\Pr(B_i^{Win} | v = \theta_i) \Pr(v = \theta_i) \right) = \\ \Pr(v = \theta_i, \text{ for some } i = 1,..,N) - \Pr(v = \theta_i, \text{ for exactly one } i = 1,..,N \text{ and there is a tie}) \end{gathered}$$

As shown in the proof of Proposition 3, in the equilibrium where the seller has no information over buyers' types we have $J = 1$ and the seller's payoff is:

$$\pi_S^{S,no\inf o} = \pi_S^{S,1,no\inf o} = \Pr(v = \theta_i, \text{ for some } i = 1,..,N-1) - c.$$

Hence the result is established by showing:

CLAIM **8**

$$\begin{aligned} \Pr(v &= \theta_i, \textit{ for some } i = 1,..,N-1) > \\ \Pr(v &= \theta_i, \textit{ for some } i = 1,..,N) - \Pr(v = \theta_i, \textit{ for exactly one } i = 1,..,N \textit{ and there is a tie}) \end{aligned}$$

■

Proof of Proposition 5 (missing details)

to a buyer's type, is unaffected but the buyer will receive such report less often and only when competition from other buyers is fiercer, so that his payoff is also lower. The proofs of this and the next two claims involve some straightforward computations and are then relegated to Appendix A.

[32] See the proof of Claim 6 in Appendix A.

1. It remains here to verify that, when the disinterested trader is the monopolist seller of information, the change in the auction revenue of the owner of the object if he were to enter the market for the sale of information is zero. Entry only changes the bidding behavior of the single buyer (say B_N) who in the monopoly equilibrium is uninformed and in the case of entry also gets a truthful report. Thus the changes in auction revenue are limited to the states where the bid of B_N is pivotal, i.e. where only one of the first $N-1$ buyers likes the object and B_N, by becoming informed, raises the price from $1/K$ to 1 if he learns that he likes the object and lowers it to 0 if he learns that he doesn't. The expected difference in revenue of the owner of the object is then

$$(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\frac{1}{K}\left(1-\frac{1}{K}\right)+(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-2}\frac{K-1}{K}\left(0-\frac{1}{K}\right)=0,$$

always zero. Note that the same is true for the duopoly equilibrium where two potential buyers sell information at a zero price since in that case all buyers are fully informed.

2. In the text we claimed that:

$$\frac{1}{K}\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right)>c^0=\frac{1}{K^2\left(N-2\right)}.$$

It is immediate to see that this inequality is always satisfied since it is equivalent to

$$\frac{K\left(N-2\right)-1}{K\left(N-2\right)}>\left(\frac{K-1}{K}\right)^{N-1}$$

and

$$\frac{K\left(N-2\right)-1}{K\left(N-2\right)}>\frac{K-1}{K}>\left(\frac{K-1}{K}\right)^{N-1}.$$

This completes the proof of the claim in part 2. of the Proposition when $c<c^0$. When $c\geq c^0$ the change in the auction revenue of the owner of the good if he were to enter (when a potential buyer is an information monopolist) is positive (goes from $1/K$ to 1) when the directly informed buyer and at least another buyer like the object and negative (goes from $1/K$ to 0) when only one directly or indirectly informed buyer likes the object. Thus the expected change in revenue is lower than the cost of entry (and thus entry is not profitable) if:

$$\frac{1}{K}\left(1-\left(\frac{K-1}{K}\right)^{N-1}\right)\left(1-\frac{1}{K}\right)-(N-1)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K}<c.$$

■

Proof of Proposition 6

Most of this proof involves routine computations which are similar in nature to those of Proposition 1 and have then been relegated to Appendix B. The only distinctive aspect is given by the following Lemma, whose proof is then reported here.

Lemma 1 *The optimal choice of the seller of information concerning the degree of differentiation of the information sold is always to have as many types of reports as the number of buyers of information.*

Proof. Suppose there are two buyers purchasing the same type of report, say l. To establish the result we show that the seller's payoff always increases by introducing some differentiation in the report sold to each of them, that is if layer l of the hierarchy is split into two adjacent ones, $l' < l''$:

1. The price paid by the seller in the auction does not change.
2. Buyers' willingness to pay for reports of a quality different from l does not vary, since the payoff of a buyer in some layer i only depends on the total number of other buyers in his same layer or above it, not on their distribution across such layers, and the first one is not affected by the split.
3. By the same argument, the willingness to pay for the lower quality report l'' is the same as the one for report l before the split, while that for report l' is strictly higher.■

Proof of Proposition 7

To establish the result we only need to show that there is always an equilibrium of the subgame with two sellers of information where they both make strictly positive profits. Suppose the first seller offers a complete hierarchy of reports, one for each of the $N-2$ buyers of information, and charges a price equal to zero for the lowest quality report and a positive price equal to half the maximal willingness to pay of a buyer for every other report. Then we claim that the best response of the second seller is to do exactly the same. If he does that, any buyer who considers purchasing a report of any quality (except the lowest one) will buy it from both sellers. Purchasing the report only from one seller does not yield in fact a priority level over all the buyers who purchase reports of lower quality, but is equivalent to purchasing the lowest quality report. Such priority level is only attained if the same type of report is bought from both sellers. By replicating the strategy of the first seller, the second seller shares so all buyers with him and obtains a positive revenue from the sale of information, approximately equal to half the monopolist revenue, and a positive payoff from the auction (as he can get the object at a zero price whenever he likes it and the other seller does not like it). Any other strategy meant to attract buyers only to the second seller is clearly less profitable.

Having shown that the profits of the two sellers are strictly positive it follows that, provided c is not too high, so will be their total payoff, net of c.■

Proof of Proposition 8

If the claim does not hold there must be at least two messages m' and m'' such that $\Pr(k = i|m') = 1 = \Pr(k = i|m'')$ and $\Pr(q = H|m') > \frac{1}{2} > \Pr(q = H|m'')$. That is, both m' and m'' truthfully reveals the variety of the object is i and, in addition, they reveal something concerning the quality of the object. In this situation the owner of the good could induce a higher bid from the Se buyers, and hence increase his profits[33], by always announcing m' rather than m'', without affecting the bid of the In types, who do not care about quality. Since the seller of information chooses always to lie when he can profit by doing so, m'' would never be sent in equilibrium, a contradiction.

This establishes the first part of the claim. It is then immediate to see that, if $\Pr(k = i|m) = 1$ for some $i \in \mathcal{K}$ and $\Pr(q = H|m) = \frac{1}{2}$, when $V > 2$ all Se buyers who like variety i always bid more for the good than In buyers who like the same variety, and vice versa when $V < 2$. ■

[33]More precisely, the seller's profits strictly increase if $\Pr(q = H|m')V > 1$. When this condition does not hold the properties of the equilibrium are the same as when $\Pr(q = H|m') = \Pr(q = H|m'') = 1/2$.

A Appendix A: Further proofs (not for publication)

Proof of Claim 7

$$\begin{aligned} \Pr(\theta_i &= v|m_S = \theta_i) = \\ \Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there is a tie}) \Pr(\text{there is a tie}) + \\ + \Pr(\theta_i &= v|m_S = \theta_i \ \cap \ \text{there are no ties}) \Pr(\text{there are no ties}). \end{aligned}$$

Furthermore, it is clear from the above reporting strategy that both

$$\begin{aligned} \Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there is a tie}) > 1/K, \\ \Pr(\theta_i &= v|m_S = \theta_i \ \cap \ \text{there are no ties}) > 1/K. \end{aligned}$$

From this it follows that the optimal bidding strategy of an indirectly informed buyer is:

$$b_i(m_S = \theta_i) = \Pr(\theta_i = v|m_S = \theta_i \cap \text{ there is a tie}), \tag{A.1}$$

since in the event in which there are no ties the buyer knows he is the only one buyer to receive a message equal to his type, hence the highest bid among all other buyers is the one of the uninformed buyer, $1/K$. Thus if the buyer knew he were in such event, any bid greater than $1/K$ would be optimal as it yields him the object. On the other hand, in the event where there is a tie, the value of the object for the buyer is $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie). Thus if the buyer knew he were in such event his optimal bid would be precisely $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie). Now the buyer does not know which of these two events is true when he makes his bid. However, a bid equal to $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie) is an optimal bid if the buyer knew which if the two events were true and can be made without knowing this.

The seller's reporting strategy in (31) and the bidding strategy of the indirectly informed buyers in (A.1) imply that in equilibrium, in every state, at least one such buyer i will receive a message $m_S = \theta_i$ and hence make a bid $b_i > 1/K$, so that the uninformed buyer never gains the object: $\Pr(B_N^{Win}) = 0$ as claimed.■

We establish below a slightly stronger result than Claim 6:

Claim **9** *Both when $J = 1$ and when $J = 0$, for any who purchases information it is optimal to truthfully report is type to the seller (given that all other buyers do that and the seller follows the reporting strategy in (31)*

Proof. Let $J = 1$. if B_1 lies about his own type to the seller (L), the above reporting strategy of the seller implies that B_1 can only receive a report $m_S = \theta_1$ if there exists at least one other buyer $j = 2, .., N-1$ such that $\theta_j = \theta_1$. Hence

$$\begin{aligned}
\Pr(\theta_1 &= v|m_S = \theta_1; L) = \Pr(\theta_1 = v|m_S = \theta_1 = \theta_j \text{ some } j; L) \qquad \text{(A.2)}\\
\Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \cap \text{ there is a 'reported' tie;L}) \Pr(\text{there is a 'reported' tie}) +\\
+ \Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \ \cap \text{ there are no 'reported' ties;L}) \Pr(\text{there are no 'reported' ties}).
\end{aligned}$$

Furthermore, notice that:

$$\begin{aligned}
\Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \cap \text{ there is a 'reported' tie;L}) = \qquad \text{(A.3)}\\
\Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there is a tie}),\\
\Pr(\theta_1 &= v|m_S = \theta_1 = \theta_j \text{ some } j \ \cap \text{ there are no 'reported' ties;L})\\
&= \Pr(\theta_i = v|m_S = \theta_i \cap \text{ there are no ties})
\end{aligned}$$

The optimal bidding strategy of B_1 is then $b_1(m_S = \theta_1; L) = \Pr(\theta_1 = v|m_S = \theta_1)$.

On this basis we can compare the payoff of B_1 if he reports truthfully his type

$$\begin{aligned}
\Pr(\text{there is no tie} \cap m_S &= \theta_1)(b_i(m_S = \theta_1) - 1/K) = \qquad \text{(A.4)}\\
\Pr(\text{there is no tie} \cap m_S &= \theta_1)(\Pr(\theta_1 = v|m_S = \theta_1 \cap \text{ there are no ties}) - 1/K)
\end{aligned}$$

to the one if he misreports his type:

$$\begin{aligned}
\Pr(m_S &= \theta_1 = \theta_j \text{ some } j; L)(\Pr(\theta_1 = v|m_S = \theta_1; L) - b_j(m_S = \theta_j)) =\\
\Pr(m_S &= \theta_1 = \theta_j \text{ some } j; L)(\Pr(\theta_1 = v|m_S = \theta_1; L) - \Pr(\theta_i = v|m_S = \theta_i \cap \text{ there is a tie}))
\end{aligned}$$

which using (A.2) and (A.3) becomes:

$$\begin{aligned}
&= \Pr(m_S = \theta_1 = \theta_j \text{ some } j; L) \qquad \text{(A.5)}\\
\times \Pr(\text{there are no 'reported' ties}) \max\{\Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there are no ties})\\
- \Pr(\theta_i &= v|m_S = \theta_i \cap \text{ there is a tie}); 0\}.
\end{aligned}$$

Comparing (A.4) and (A.5), and noticing that Pr(there are no 'reported' ties) = Pr(there are no ties), $\Pr(m_S = \theta_1 = \theta_j \text{ some } j; L) < \Pr(m_S = \theta_1)$ and recalling that $\Pr(\theta_i = v|m_S = \theta_i \cap$ there is a tie$) > 1/K$ establishes the result.

We establish next that the same result holds also when $J = 0$. In that case buyers would still want to report truthfully their type. This follows from the fact that now, with no uninformed traders, the buyers who report truthfully their type make a bid equal to $\Pr(\theta_i = v|m_S = \theta_i)$ and, by essentially the same argument as above, $\Pr(\theta_1 = v|m_S = \theta_1) = \Pr(\theta_1 = v|m_S = \theta_1; L)$, i.e. lying does not change the information contained in a report equal to the buyer's own type. Since that, in the even of a lie, only happens if there is at least one other buyer who receives that report, who will then make that bid, the expected payoff by lying is zero.■

Proof of Claim 8

$$\begin{aligned}
\Pr(v &= \theta_i,\ \text{for exactly one } i = 1,..,N \text{ and there is a tie}) > \\
\Pr(v &= \theta_i,\ \text{for some } i = 1,..,N) - \Pr(v = \theta_i,\ \text{for some } i = 1,..,N-1) = \\
&= \left(1 - \left(\frac{K-1}{K}\right)^N\right) - \left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) = \left(\frac{K-1}{K}\right)^{N-1}\left(\frac{1}{K}\right).
\end{aligned}$$

Note that

$$\begin{aligned}
\Pr(v &= \theta_i,\ \text{for exactly one } i = 1,..,N \text{ and there is a tie}) = \\
&= 1 - \Pr(\text{no ties}) - \Pr(\text{tie on } v) \\
&= 1 - \left[\left(\frac{K-1}{K}\right) \cdot ... \left(\frac{K-(N-1)}{K}\right)\right] - \left[1 - \left(\frac{K-1}{K}\right)^N - \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1}\right] \\
&= \left(\frac{K-1}{K}\right)^N + \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1} - \left[\left(\frac{K-1}{K}\right) \cdot ... \left(\frac{K-(N-1)}{K}\right)\right] > \\
&\left(\frac{K-1}{K}\right)^N + \frac{N}{K}\left(\frac{K-1}{K}\right)^{N-1} - \left(\frac{K-1}{K}\right)^{N-1},
\end{aligned}$$

which is clearly greater than $\left(\frac{K-1}{K}\right)^{N-1}\left(\frac{1}{K}\right)$.■

Proof of Proposition 6

With the message structure, there are again no out-of-equilibrium messages. Thus, we can find the beliefs of the uninformed buyers using Bayes' rule in all cases. Let n_l be the number of buyers in layer l and N_l the total number of buyers in layers 1 through l plus the seller of information $B_1 : N_l = \sum_{j=0}^{l} n_j$, where we adopt the convention $n_0 = 1$.

The beliefs of buyer B_j purchasing a report of type l are:

1. When B_j receives a message $m_l = \theta_j$ he knows for sure he likes the object. That is[34]

$$\Pr(v = \theta_j|m_l = \theta_j) = 1$$

[34] Even though a buyer receiving message m_l also receives messages $m_j, j > l$, given the nested structure of the information these reports have no additional informational value and can be ignored in the derivation of the conditional expectations.

2. On the other hand, when B_j receives a message $m_l \neq \theta_j$, this may happen either because $v = m_l$, or because the sender or somebody in an earlier layer $t < l$ likes the object, in which case the sender does not tell the truth and randomizes over the types which are absent from the population of buyers of information, including B_1. Given this:

$$\Pr(m_l = \theta_j) = \left(\frac{K-1}{K}\right)^{N_{l-1}} \frac{1}{K}, \quad \Pr(m_l \neq \theta_j) = 1 - \left(\frac{K-1}{K}\right)^{N_{l-1}} \frac{1}{K}$$

and

$$\begin{aligned} \Pr(v &= \theta_j | m_l \neq \theta_j) = \frac{\Pr(v = \theta_j \cap m_l \neq \theta_j)}{\Pr(m_l \neq \theta_j)} = \frac{\Pr(v = \theta_j) - \Pr(v = \theta_j \cap m_l = \theta_j)}{\Pr(m_l \neq \theta_i)} \\ &= \frac{\frac{1}{K} - \left(\frac{K-1}{K}\right)^{N_{l-1}} \frac{1}{K}}{1 - \left(\frac{K-1}{K}\right)^{N_{l-1}} \frac{1}{K}} = \frac{1 - \left(\frac{K-1}{K}\right)^{N_{l-1}} \frac{1}{K}}{K - \left(\frac{K-1}{K}\right)^{N_{l-1}} \frac{1}{K}} \end{aligned}$$

The beliefs of buyers who do not purchase information nor acquire it directly are then unchanged:

$$\Pr(v = \theta_j) = \frac{1}{K}, \quad \Pr(v \neq \theta_j) = \frac{K-1}{K}$$

Behavior in the auction We begin again by the last stage of the subgame, by characterizing the agents' behavior in the auction (still under the restriction to "truthful bidding strategies").

1. When an agent B_j receives a message $m_l = \theta_j$ it is clear that the optimal bid is equal to his posterior beliefs about the valuation of the object. That is his bid is equal to 1

2. When an agent B_j receives a message $m_l \neq \theta_j$, the optimal bid is equal to $\Pr(v = \theta_j | m_i \neq \theta_j)$. An agent may receive this signal for two reasons. Either $v = \theta_i$ for some B_i in an earlier layer, in which case he cannot win the object in the auction as agent B_i, who is better informed will make a higher bid, or $v \neq \theta_i$ for all B_i in earlier layers and $v \neq \theta_j$, in which case he may indeed win the auction by bidding a positive price. But that will entail a negative surplus. This implies that the optimal bid is zero when $m_l \neq \theta_j$, once the information conveyed by winning the auction is taken into account (affiliation).

3. Finally, the buyers who do not listen to the reports of the directly informed seller do not suffer from an affiliation problem. Thus, the optimal bid in that case is: $\Pr(v = \theta_j) = \frac{1}{K}$.

Behavior in the message game Next we consider agents' behavior in the message game. Given the reporting strategy described in equations (12) and (13) of the seller of information, we now show that the optimal reporting strategy of every buyer who is purchasing the information is to truthfully report his type. This can be shown in an almost identical way as in the case of homogeneous quality of information. Next we show that the reporting strategy of the seller of information is also optimal for this agent.

Optimality of truthful reporting for the buyers of information First of all, notice that a change in the message strategy of a buyer of information can only change the outcome of the auction, not the price paid for information. A deviation by the buyer of information consists in reporting anything other than his type. We divide this discussion in two cases.

1. Let the buyer of information be agent B_j, and suppose he purchased report l. If $v = \theta_1$ or $v = \theta_i$, for some buyer of information B_i in some layer $t < l$, either B_1 or B_i will bid 1 in the auction, no matter what is the report of B_j. Hence there is no possibility for B_j to obtain any extra surplus by misreporting his type.

2. If $v \neq \theta_1$ and $v \neq \theta_i$ for all buyers of information B_i in any layer $t < l$, then (13) prescribes that $m_t = v$ for all $t = 1, .., l$, no matter what is the report sent by agents who purchase report l. The reports of agents in layer l only affect the reports sent by B_1 to agents in layers $t' > l$. Under truthful reporting by B_j all buyers in layers $t > l$ receive a message inducing them to bid zero when $v = \theta_j$. The effect of B_j's misreporting his type is that buyers in layers $t > l$ will receive a message which will induce them either to bid zero, or a positive amount, thus lowering, at least weakly B_j expected gains from the auction. So misreporting is not optimal for B_j.

Optimality of the message for the seller of information As for the buyers of information, a change in the seller's reporting strategy has no effect on the revenue from the sale of information, only on the outcome of the auction.

1. When $v = \theta_1$, i.e. the seller of information likes the object, he can deviate and send, for some $l \in \{1, .., L\}$ a message $m_l = \theta_k$ for some buyer B_k purchasing a report of some type. In this case, the bid of B_k will equal 1, and the seller of information has to pay more for the object than if he had followed the reporting strategy (12) and (13), so the deviation is clearly not optimal.

2. When $v \neq \theta_1$, the seller is not interested in the object. Since any deviation from the messages prescribed by his reporting (12) and (13) only changes the outcome in the auction, in which he is not interested, the seller can never gain from such deviation.

Sale of information

Payoffs for the monopolist with no individual without information The single informed trader acts as a monopolist in the market for information. The maximal rent he can extract from the $N-1$ uninformed buyers purchasing information from him, for any given layer structure, is determined by comparing the payoff a buyer can get by acquiring the information of the quality associated to the layer he is in with the alternative payoff he could get by not purchasing the information.

To evaluate the payoff of a buyer in layer l we need to distinguish the case (i) where in layer l there is a single buyer from the case (ii) where in that layer there is more than a single buyer. In case (i) the buyer in layer l will always get the commodity when he likes it and no other buyer in the layers above likes it (an event with probability $\left(\frac{K-1}{K}\right)^{N_l-1}\frac{1}{K}$), and the price he will pay will be the second highest bid after his, the bid made by the bidders with less information[35], i.e. in the lowest layer, given by 0. On the other hand in case (ii) the same is true when the buyer likes the object and no other buyer in the layers above *as well as in his own layer* l likes it (an event with probability $\left(\frac{K-1}{K}\right)^{N_l-1}\frac{1}{K}$). When some other buyer in layer l likes the object, the buyer will get the object with some probability but will pay an amount equal to his valuation so that his payoff will be zero. In case (i) we have $N_l = N_{l-1}+1$; hence in both cases the payoff in the auction for a buyer in layer l is given by the following expression:

$$\left(\frac{K-1}{K}\right)^{N_l-1}\frac{1}{K}$$

If we add to this the price paid to acquire the information, we obtain the expression of the total payoff to a buyer B_i of acquiring information in layer l of a chain of length L (at a price p_l), when all the N players are connected to a unique buyer, is

$$\pi_{B_i} = \left(\frac{K-1}{K}\right)^{N_l-1}\frac{1}{K} - p_l.$$

The buyer's alternative payoff if he chooses not to buy any information and hence remain unconnected, is given by $\pi_u = \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K}$. Alternatively, the buyer could also choose to

[35]Note that in this case the buyer will be the only one receiving a message equal to his own type.

acquire directly the information, in the third and last stage, as a cost c, in which case his payoff would be $\pi_c = \left(\frac{K-1}{K}\right)^{N-1} \frac{1}{K} - c$. Note that both π_u and π_c are independent of the number L of layers. Notice that $\max\{\pi_u, \pi_c\} = \pi_u = \left(\frac{K-1}{K}\right)^{N-1} \frac{1}{K}$

The maximal rent the monopolist selling the information can extract from buyer B_i, and hence the maximal value of the price he can charge, is thus given by

$$p_l = \frac{1}{K}\left(\frac{K-1}{K}\right)^{N_l-1} - \max\{\pi_u, \pi_c\} = \frac{1}{K}\left(\left(\frac{K-1}{K}\right)^{N_l-1} - \left(\frac{K-1}{K}\right)^{N-1}\right) \tag{A.6}$$

The total payoff of B_1, the informed buyer, from acquiring the commodity when $v = \theta_1$ and from selling the information to the other buyers, is thus given by:

$$\pi_{B_1} = \frac{1}{K} + \sum_{l=1}^{L} n_l p_l - c$$

where p_l is as in expression (A.6). Note that the second term depends on the distribution of buyers across layers, i.e. on all the values $n_l, l = 0, 1, .., L$.

To obtain the optimal distribution of buyers across the layers we need then consider only its effects on the revenue from the sale of information.

$$\begin{aligned} \pi_{B_1} &= \frac{1}{K} + \sum_{l=1}^{L} n_l \frac{1}{K}\left(\left(\frac{K-1}{K}\right)^{N_l-1} - \left(\frac{K-1}{K}\right)^{N-1}\right) - c = \\ &= \frac{1}{K}\left(1 + \sum_{l=1}^{L} n_l \left(\left(\frac{K-1}{K}\right)^{N_l-1} - \left(\frac{K-1}{K}\right)^{N-1}\right)\right) - c \end{aligned} \tag{A.7}$$

Lemma 2 *The optimal distribution is to create as many layers as remaining players.*

Proof 1 *Same as Lemma 1*

We denote then the payoff of the first buyer $\pi_{B_1}(2)$ to allude to this possible equilibrium configuration. Notice that we are thus assuming $n_l = 1$ for all l. So from (A.7) we have

$$\begin{aligned} \pi_{B_1}(2) &= \frac{1}{K}\left(1 + \sum_{l=1}^{L} n_l \left(\left(\frac{K-1}{K}\right)^{N_l-1} - \left(\frac{K-1}{K}\right)^{N-1}\right)\right) - c \\ &= 1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c \end{aligned}$$

Payoffs for the monopolist with one individual without information In this case, again, the maximal rent the monopolist can extract from the $N-2$ buyers purchasing information from him, for any given layer structure, is determined by comparing the payoff a buyer can get by acquiring the information of the quality associated to the layer he is in with the alternative payoff he could get by not purchasing the information.

To evaluate then the payoff of a buyer in layer l we need to distinguish the case (i) where in layer l there is a single buyer from the case (ii) where in that layer there is more than a single buyer. In case (i) the buyer in layer l will always get the commodity when he likes it and no other buyer in the layers above likes it (an event, as we said, with probability $\left(\frac{K-1}{K}\right)^{N_l-1}\frac{1}{K}$), and the price he will pay will be the second highest bid after his, the bid made by the bidders with less information, i.e. the uninformed buyer, given by $\frac{1}{K}$. On the other hand in case (ii) the same is true when the buyer likes the object and no other buyer in the layers above *as well as in his own layer* l likes it (an event with probability $\left(\frac{K-1}{K}\right)^{N_l-1}\frac{1}{K}$). When some other buyer in layer l likes the object the buyer will get the object with some probability but will pay am amount equal to his valuation so that his payoff will be zero. In case (i) we have $N_l = N_{l-1}+1$; hence in both cases the payoff in the auction for a buyer in layer l is given by the following expression:

$$\left(\frac{K-1}{K}\right)^{N_l-1}\frac{1}{K}\left(1-\frac{1}{K}\right) = \left(\frac{K-1}{K}\right)^{N_l}\frac{1}{K}$$

If we add to this the price paid to acquire the information, we obtain the expression of the total payoff to a buyer B_i of acquiring information in layer l of a chain of length L (at a price p_l), when all the N players are connected to a unique buyer, is

$$\pi_{B_i} = \left(\frac{K-1}{K}\right)^{N_l}\frac{1}{K} - p_l.$$

The buyer's alternative payoff if he chooses not to buy any information and hence remain unconnected, is given by $\pi_u = 0$. Alternatively, the buyer could also choose to acquire directly the information, in the third and last stage, as a cost c, in which case his payoff would be $\pi_c = \left(\frac{K-1}{K}\right)^{N-2}\frac{1}{K}\left(1-\frac{1}{K}\right) - c = \left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} - c$. Note that both π_u and π_c are independent of the number L of layers and that $\max\{\pi_u, \pi_c\} = \max\left\{\left(\frac{K-1}{K}\right)^{N-1}\frac{1}{K} - c, 0\right\}$

The maximal rent the monopolist selling the information can extract from buyer B_i, and hence the maximal value of the price he can charge, is thus given by

$$\begin{aligned} p_l &= \frac{1}{K}\left(\frac{K-1}{K}\right)^{N_l} - \max\{\pi_u, \pi_c\} && \text{(A.8)} \\ &= \min\frac{1}{K}\left(\left(\frac{K-1}{K}\right)^{N_l}, \left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c\right) && \text{(A.9)} \end{aligned}$$

The total payoff of B_1, the informed buyer, from acquiring the commodity when $v = \theta_1$ and from selling the information to the other buyers, is thus given by:

$$\pi_{B_1}(1) = \frac{1}{K}\left(1 - \frac{1}{K}\right) + \sum_{l=1}^{L} n_l p_l - c,$$

where p_l is as in expression (A.8). Note that the second term also depends on the distribution of buyers across layers, i.e. on all the values $n_l, l = 0, 1, .., L$.

To obtain the optimal distribution of buyers across the layers we need then consider only its effects on the revenue from the sale of information.

$$\begin{aligned}
\pi_{B_1}(1) &= \frac{1}{K}\left(1 - \frac{1}{K}\right) + \sum_{l=1}^{L} n_l \min \frac{1}{K}\left(\left(\frac{K-1}{K}\right)^{N_l}, \left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c\right) - c = \\
&= \frac{1}{K}\left(1 - \frac{1}{K} + \sum_{l=1}^{L} n_l \min\left(\left(\frac{K-1}{K}\right)^{N_l}, \left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c\right)\right) - c
\end{aligned}$$

Lemma **3** *The optimal distribution is to create as many layers as remaining players.*

Proof 2 *Same as Lemma 1*

When does the price change from $\left(\frac{K-1}{K}\right)^{N_l}$ **to** $\left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c$ $\quad \left(\frac{K-1}{K}\right)^{N_l-1} \geq \left(\frac{K-1}{K}\right)^{N_l-1} - \left(\frac{K-1}{K}\right)^{N-1} + c$ if and only if

$$\left(\frac{K-1}{K}\right)^{N-1} \geq c$$

When does the monopolist prefer to have no unconnected players?

1. Assume first that
$$\left(\frac{K-1}{K}\right)^{N-1} \geq c$$

   Then the payoff under 1 is

$$\begin{aligned}
\pi_{B_1}(1) &= \frac{1}{K}\left(1 - \frac{1}{K} + \sum_{l=1}^{L} n_l \left(\left(\frac{K-1}{K}\right)^{N_l} - \left(\frac{K-1}{K}\right)^{N-1} + c\right)\right) - c \\
&= \left(\frac{K-1}{K}\right) - \left(\frac{K-1}{K}\right)^{N} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} + (N-2)\frac{c}{K} - c
\end{aligned}$$

   We also know from above that

$$\pi_{B_1}(2) = 1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-2)\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$$

So $\pi_{B_1}(2) \geq \pi_{B_1}(1)$ iff

$$1 - \left(\frac{K-1}{K}\right)^{N-1} \geq \left(\frac{K-1}{K}\right) - \left(\frac{K-1}{K}\right)^{N} + (N-2)\frac{c}{K}$$

So in fact the monopolist prefers to have no unconnected players provided c is low enough so that

$$\begin{aligned}\frac{1}{K}\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) &\geq (N-2)\frac{c}{K} \\ 1 - \left(\frac{K-1}{K}\right)^{N-1} &\geq (N-2)c\end{aligned}$$

2. For

$$\left(\frac{K-1}{K}\right)^{N-1} < c$$

We are in the regime where

$$\begin{aligned}\pi_{B_1}(1) &= \frac{1}{K}\left(1 - \frac{1}{K} + \sum_{l=1}^{N-2}\left(\frac{K-1}{K}\right)^{l+1}\right) - c \\ &= \left(\frac{K-1}{K}\right)\left(\frac{1}{K} + \left(\frac{K-1}{K}\right) - \left(\frac{K-1}{K}\right)^{N-1}\right) - c\end{aligned}$$

Thus

$$\pi_{B_1}(1) = \left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) - c \tag{A.10}$$

and since

$$\pi_{B_1}(2) = 1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-2)\,\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} - c$$

So $\pi_{B_1}(1) \geq \pi_{B_1}(2)$ iff

$$\begin{aligned}\left(\frac{K-1}{K}\right)\left(1 - \left(\frac{K-1}{K}\right)^{N-1}\right) &\geq 1 - \left(\frac{K-1}{K}\right)^{N-1} - (N-2)\,\frac{1}{K}\left(\frac{K-1}{K}\right)^{N-1} \\ (N-1) &\geq \left(\frac{K}{K-1}\right)^{N-1} = \left(1 + \frac{1}{K-1}\right)^{N-1}\end{aligned}$$

but notice that

$$
\begin{aligned}
\left(1+\frac{1}{K-1}\right)^{N-1} &= \sum_{t=0}^{N-1}\binom{N-1}{t}\frac{1}{(K-1)^t} \\
&= 1+\sum_{t=1}^{N-1}\frac{(N-1)(N-2)...(N-t)}{(1\cdot 2\cdot ...\cdot t)\,(K-1)^t} \\
&\leq 1+\frac{(N-1)}{(K-1)}+\sum_{t=2}^{N-1}\frac{1}{2}\frac{(N-1)^t}{(K-1)^t} \\
&\leq 2+\frac{(N-2)}{2}=\frac{(N+2)}{2}
\end{aligned}
$$

but since

$$
N\geq 4 \Longleftrightarrow (N-1)\geq\frac{(N+2)}{2}, \text{and } \frac{(N+2)}{2}\geq\left(\frac{K}{K-1}\right)^{N-1}
$$

then $\pi_{B_1}(1)\geq\pi_{B_1}(2)$. So in this regime, 1 always dominates 2.

The previous considerations show that both 1 and 2 can be preferred for different values of c, and overall the condition that determines when 2 is preferred by the monopolist is:

$$
1-\left(\frac{K-1}{K}\right)^{N-1}\geq (N-2)c \tag{A.11}
$$

This completes the proof of Proposition 6